SOLID MECHANICS
DYNAMICS

MOTION - DISPLACEMENT, VELOCITY AND ACCELERATION

On completion of this tutorial you should be able explain and use equations for linear and angular motion.

Contents

1. Introduction

The movement of things is very important in dynamics. This involves things that move in straight lines (linear motion) and things that move in circles (angular motion). This applies to many forms of mechanism and machinery or structures that move. Understanding the dynamics of motion means you must understand displacement, velocity and acceleration and the laws governing them. Let's start with linear motion.

2. *Linear Motion*

2.1 Movement or Displacement

This is the distance travelled by an object and is usually denoted by x or s. Units of distance are metres.

2.2 Velocity

This is the distance moved per second or the rate of change of distance with time. Velocity is movement in a known direction so it is a vector quantity. The symbol is v or u and it may be expressed in calculus terms as the first derivative of distance with respects to time so that:

$$\mathbf{v = \frac{dx}{dt} \quad or\ u = \frac{ds}{dt} \quad m/s}$$

2.3. Speed

This is the same as velocity except that the direction is not known and it is not a vector and cannot be drawn as such.

2.4. Average Speed or Velocity

When a journey is undertaken in which the body speeds up and slows down, the average velocity is defined as:

$$\mathbf{average\ velocity = \frac{Total\ Distance\ Moved}{Time\ Taken}}$$

2.5. Acceleration

When a body slows down or speeds up, the velocity changes and acceleration or deceleration occurs. Acceleration is the rate of change of velocity and is denoted with a. In calculus terms it is the first derivative of velocity with time and the second derivative of distance with time such that:

$$\mathbf{a = \frac{dv}{dt} \quad = \frac{d^2x}{dt^2}\ m/s^2}$$

Note that all bodies falling freely under the action of gravity experience a downwards acceleration of 9.81 m/s^2.

WORKED EXAMPLE No. 1

A vehicle accelerates from 2 m/s to 26 m/s in 12 seconds. Determine the acceleration. Also find the average velocity and distance travelled.

SOLUTION

$$a = \frac{\Delta v}{t} = \frac{(26-2)}{12} = 2\frac{m}{s^2} \quad \text{AverageVelocity} = \frac{(26+2)}{2} = 14\ m/s$$

Distance travelled = 14 × 12 = 168 m

SELF ASSESSMENT EXERCISE No. 1

1. A body moves 5 000 m in 25 seconds. What is the average velocity?
(Answer 200 m/s)

2. A car accelerates from rest to a velocity of 8 m/s in 5 s. Calculate the average acceleration.
(Answer 1.6 m/s^2)

3. A train travelling at 20 m/s decelerates to rest in 40 s. What is the acceleration?
(Answer -0.5 m/s^2)

2.6. Graphs

It is very useful to draw graphs representing movement with time. Much useful information may be found from the graph.

Distance - Time Graphs

Graph A shows constant distance at all times so the body must be stationary.

Graph B shows that every second, the distance from start increases by the same amount so the body must be travelling at a constant velocity.

Graph C shows that every passing second, the distance travelled is greater than the one before so the body must be accelerating.

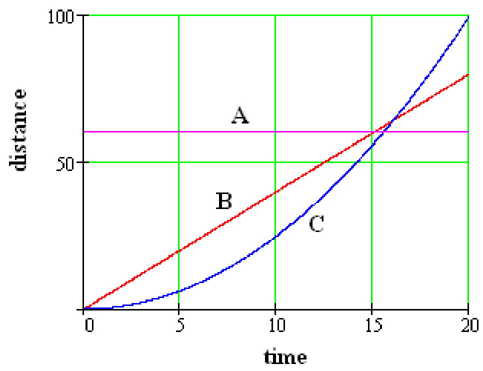


Figure 1

Velocity - Time Graphs

Graph B shows that the velocity is the same at all moments in time so the body must be travelling at constant velocity.

Graph C shows that for every passing second, the velocity increases by the same amount so the body must be accelerating at a constant rate.

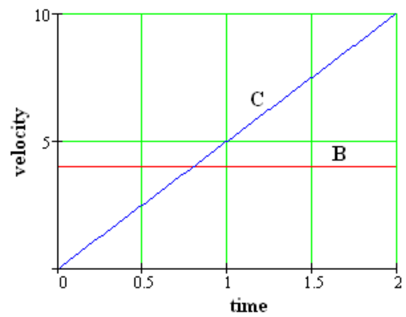


Figure 2

2.7 Significance of the Area under a Velocity-Time Graph

The average velocity is the average height of the v - t graph. By definition, the average height of a graph is the area under it divided by the base length.
Average velocity = total distance travelled/time taken = area under graph/base length

Since the base length is also the time taken it follows that the area under the graph is the distance travelled. This is true what ever the shape of the graph. When working out the areas, the true scales on the graphs axis are used.

WORKED EXAMPLE No. 2

Find the average velocity and distance travelled for the journey depicted on the graph. Also find the acceleration over the first part of the journey.

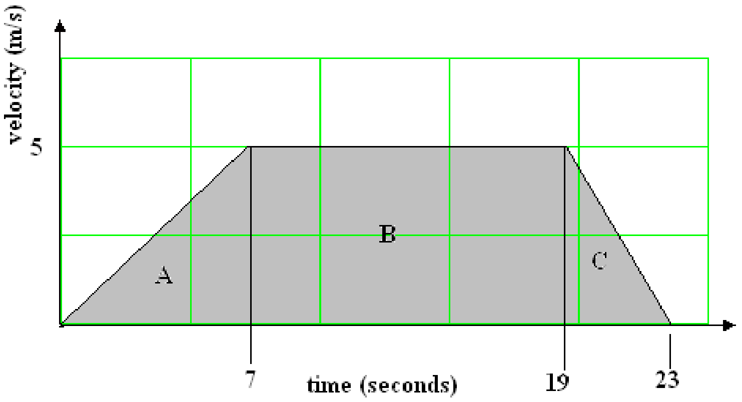


Figure 3

SOLUTION

Total Area under graph = A + B + C

Area A = 5 × 7/2 = 17.5 (Triangle)
Area B = 5 × 12 = 60 (Rectangle)
Area C = 5 × 4/2 = 10 (Triangle)
Total area = 17.5 + 60 + 10 = 87.5

The units resulting for the area are m/s × s = m

Distance travelled = 87.5 m   Time taken = 23 s   Average velocity = 87.5/23 = 3.8 m/s

Acceleration over part A = change in velocity/time taken = 5/7 = 0.714 m/s^2.

SELF ASSESSMENT EXERCISE No. 2

1. A vehicle travelling at 1.5 m/s suddenly accelerates uniformly to 5 m/s in 30 seconds. Calculate the acceleration, the average velocity and distance travelled.
(Answers 0.117 m/s^2, 3.25 m/s and 97.5 m)

2. A train travelling at 60 km/h decelerates uniformly to rest at a rate of 2 m/s^2. Calculate the time and distance taken to stop.
(Answers 8.33 s and 69.44 m)

3. A shell fired in a gun accelerates in the barrel over a length of 1.5 m to a velocity of 220 m/s. Calculate the time taken to travel the length of the barrel and the acceleration of the shell.
(Answers 0.01364 s and 16 133 m/s^2)

2.8. Standard Formulae

Consider a body moving at constant velocity u. Over a time period t seconds it accelerates from u to a final velocity v.

The graph looks like this.

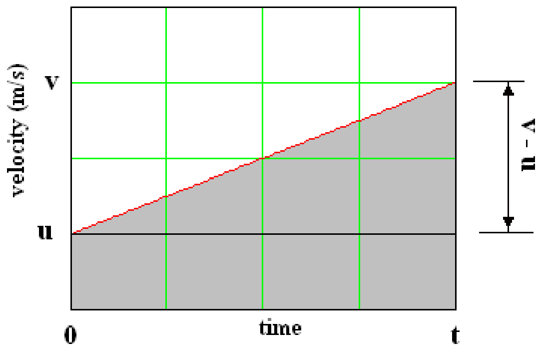


Figure 4

Distance travelled = s
s = area under the graph

$$\mathbf{s = ut + \frac{(v-u)t}{2} \ldots\ldots\ldots (1)}$$

Multiply out

$$s = ut + \frac{vt}{2} - \frac{ut}{2} = \frac{vt}{2} + \frac{ut}{2} = \frac{t}{2}(v+u)$$

The acceleration = a

$$a = \frac{(v-u)}{t} \text{ hence } (v-u) = at$$

Substituting this into equation (1) gives:

$$\mathbf{s = ut + \frac{at^2}{2} \ldots\ldots\ldots (2)}$$

Since v = u + the increase in velocity v = u + at
Squaring we get:

$$\mathbf{v^2 = u^2 + 2a\left[\frac{at^2}{2} + ut\right] = u^2 + 2as \ldots\ldots\ldots (3)}$$

These are the three equations commonly used to solve problems such as those following.

WORKED EXAMPLE No. 3

A missile is fired vertically with an initial velocity of 400 m/s. It is acted on by gravity. Calculate the height it reaches and the time taken to go up and down again.

SOLUTION

u = 400 m/s   v = 0   a = -g = -9.81 m/s^2

$v^2 = u^2 + 2as$ $0 = 400^2 + 2(-9.81)s$ s = 8 155 m

$$s = 8\,155 = \frac{t}{2}(v+u) \qquad t = \frac{8\,155 \times 2}{(400+0)} = 40.77 \text{ s}$$

To go up and down takes twice as long. t = 81.54 m/s

WORKED EXAMPLE No. 4

A lift is accelerated from rest to 3 m/s at a rate of 1.5 m/s^2. It then moves at constant velocity for 8 seconds and then decelerates to rest at 1.2 m/s^2. Draw the velocity - time graph and deduce the distance travelled during the journey. Also deduce the average velocity for the journey.

SOLUTION

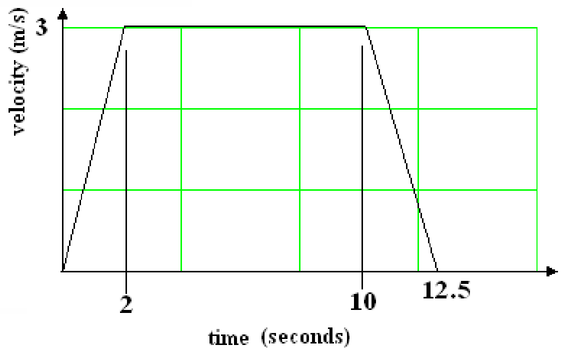


Figure 5

The velocity - time graph is shown.

The first part of the graph shows uniform acceleration from 0 to 3 m/s.

The time taken is given by $t_1 = 3/1.5 = 2$ seconds.

The distance travelled during this part of the journey is $x_1 = 3 \times 2/2 = 3$ m

The second part of the journey is a constant velocity of 3 m/s for 8 seconds so the distance travelled is $x_2 = 3 \times 8 = 24$ m

The time taken to decelerate the lift over the third part of the journey is $t_3 = 3/1.2 = 2.5$ seconds.

The distance travelled is $x_2 = 3 \times 2.5/2 = 3.75$ m

The total distance travelled is 3 + 24 + 3.75 = 30.75 m.

The average velocity = distance/time = 30.75/12.5 = 2.46 m/s.

SELF ASSESSMENT EXERCISE No. 3

The diagram shows a distance-time graph for a moving object. Calculate the velocity. (Answer 1.82 m/s)

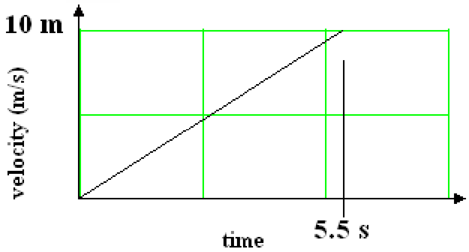


Figure 6

2. The diagram shows a velocity time graph for a vehicle. Calculate the following.

i. The acceleration from O to A. (2.857 m/s^2)
ii. The acceleration from A to B. (-2.4 m/s^2)
iii. The distance travelled. (35 m)
iv. The average velocity. (5.83 m/s)

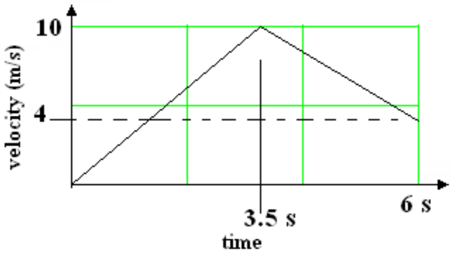


Figure 7

3. The diagram shows a velocity-time graph for a vehicle. Calculate the following.

i. The acceleration from O to A. (5 m/s^2)
ii. The acceleration from B to C. (-2 m/s^2)
iii. The distance travelled. (85 m)
iv. The average velocity. (7.08 m/s)

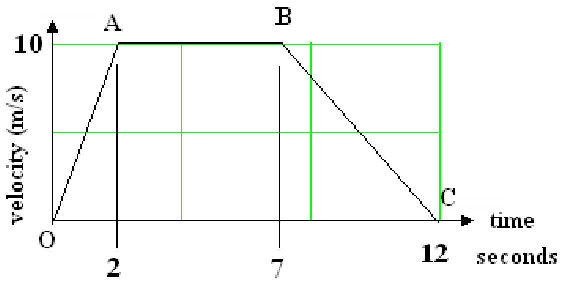


Figure 8

3. *Mass and Acceleration*

We need to study what happens when a force is applied to a body and it is free to move under the action of that force. This is an area of study called ***Dynamics*** and the forces are dynamic forces.

In this unit we will only be studying cases where the motion is along a straight line. We will not be studying what happens when the force makes a body rotate or spin but this is very important area of study.

The most important point is that if a body is free to move then a force will make it accelerate in the direction of the force. The degree of acceleration (or deceleration) depends on the inertia of the body. Inertia is really another word for mass. A good definition of mass and inertia is that it is a property of a body that enables it to resist changes in its motion. The greater the mass and hence inertia, the more difficult it is to make a body speed up or slow down or change direction. This is why a motorcycle can usually out accelerate a large vehicle, because it has a smaller mass. This is why a train takes longer to stop than a car, because it has so much inertia. The way force, mass and acceleration are linked is governed by Newton's Laws of Motion.

3.1. *Newton's Laws of Motion*

Newton studied the way that forces make bodies change their motion and came up with his three laws.

1. **A body at rest or with uniform motion will remain at rest or continue with uniform motion until it is acted on by an external force.**
2. **An external force will cause the body to accelerate or decelerate.**
3. **Every force has an equal and opposite reaction.**

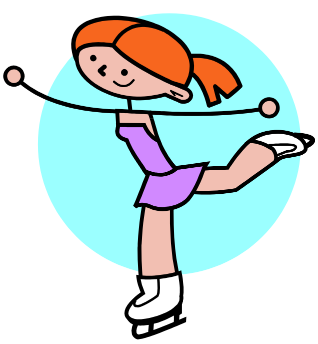
The essence of these laws is that in order to change the motion of a body you must accelerate it or decelerate it by applying a force to it.

The INERTIA of a body is the property that enables it to resist changes in its motion.

The work you need to study is mainly concerned with the second law.

Explanation

Imagine a person on an ice rink with absolutely no friction between the skates and the ice. If she was moving, she would be unable to neither slow down nor speed up. She could only change her motion if an external force was applied to her. This is an example of the first law.



In reality the external force is obtained by finding friction with the ice by digging the skates into the ice and pushing or braking. This force produces changes in the motion of the skater. Using friction to enable him to either accelerate or decelerate is an application of the second law.

Next, imagine the person stationary on the ice. In his hands he has a heavy ball. If he threw the ball away, he would move on the ice. In order to throw the ball away he must exert a force on the ball. In return, the ball exerts an equal and opposite force on the person so he moves away in the opposite direction to the ball. This is an example of the third law.

The same principles apply to a space vehicle. There is no friction in space and the only way to change the motion of a space vehicle is to eject matter from a rocket so that the reaction force acts on the vehicle and changes its motion.

The law which has the greatest significance for us is the 2^{nd} law so let's look at this in detail.

3.2 Newton's 2^{nd} Law of Motion

The most fundamental way of stating the second law is:

The IMPULSE given to a body is equal to the change in MOMENTUM.

Impulse is defined as the product of force and the time for which it is applied.

$$\textbf{Impulse} = \textbf{Force} \times \textbf{Time} = \textbf{Ft}$$

WORKED EXAMPLE No. 5

A vehicle has a force of 400 N applied to it for 20 seconds. Calculate the impulse?

SOLUTION

IMPULSE = Ft = 400 × 20 = 8000 N s

Momentum is defined as the product of mass and velocity.

$$\textbf{Momentum} = \textbf{Mass} \times \textbf{Velocity} = \textbf{m v} \quad \textbf{kg m/s}$$

WORKED EXAMPLE No. 6

A vehicle of mass 5 000 kg changes velocity from 2 m/s to 6 m/s. Calculate the change in momentum.

SOLUTION

Initial momentum = mu_1 = 5 000 × 2 = 10 000 kg m/s

Final momentum = mu_2 = 5 000 × 8 = 40 000 kg m/s

Change in momentum = 40 000 - 10 000 = 30 000 kg m/s

Rewriting the 2nd Law

From the statement Impulse = change in momentum, the second law can be written as:

$$\mathbf{Ft = \Delta mu}$$

This equation may be rearranged into other forms as follows. $F = \Delta mu/t$

If the mass is constant and since acceleration = $\Delta u/t$ this becomes $F = m\ \Delta u/t = m\ a$

Force = Mass × Acceleration

The above equation is suitable for problems involving solid bodies but for the solution of problems involving flowing liquids or gasses we use the following.

$$F = u\ \Delta m/t = \text{velocity} \times \text{mass flow rate}$$

Not used in this tutorial but we should note that in fluid flow problems like solving the forces on pipe bends and turbine blades, we use this form of the equation.

$$\mathbf{F = \Delta mu/t = \text{rate of change of momentum}}$$

WORKED EXAMPLE No. 7

A mass of 8 kg accelerates at 6 m/s^2 for 5 seconds. Calculate the following:

i. The force producing acceleration
ii. The change in velocity
iii. The change in momentum
iv. The impulse.

SOLUTION

$F = M\ a = 8 \times 6 = 48$ N
Change in velocity = at = $6 \times 5 = 30$ m/s
Change in momentum = $M\ \Delta v = 8 \times 30 = 240$ kg m/s^2
Impulse = F t = $48 \times 5 = 240$ N s

Check Impulse = change in momentum

SELF ASSESSMENT EXERCISE No. 4

1. A vehicle of mass 1 100 kg moves at 3 m/s. The brakes are applied and the vehicle reduces speed to 0.5 m/s in 45 s. Calculate the force needed.

2. A rocket of mass 200 kg in outer space moves at 360 m/s. It accelerates in a straight line by firing its motors with a force of 50 N. Calculate how long it takes to reach a velocity of 700 m/s.

4. *The Effect of Friction*

Friction is an external force that always acts to oppose motion. When a body is accelerated, the force causing acceleration is the ***Net Force.***

Net Force = Applied Force - Friction Force.

SELF ASSESSMENT EXERCISE No. 5

1. The applied force on a vehicle is 6000 N but the wind and road resistance is 2 000 N. Calculate the acceleration of the vehicle. The mass is 2 000 kg
2. Calculate the force needed to accelerate a piston of mass 0.8 kg in a cylinder at 3 m/s^2 if the resisting force is 3 N.

5. *Gravity*

All bodies exert a force of attraction on each other but this force diminishes with distance. Unless one of the bodies is very large and/or very close, the effect is very small. In the case of the Earth, it is large and we are close so it exerts a force of gravity on everything close to it such that any unrestrained body would accelerate downwards towards the centre of the Earth at 9.81 m/s^2. This is the gravitational constant "g". Galileo showed that large and small bodies fall at the same rate with his famous experiment when he dropped two cannon balls of different sizes from the leaning tower of Pisa. They hit the ground together. This experiment works for compact masses but if you dropped a spanner and a feather, the feather would float around on the air currents. Astronauts on the moon showed that in the absence of air, a spanner and a feather fall together.

Since all bodies are accelerated downwards at the same rate then the force acting on it is F = M g. In order to stop a body falling, an equal and opposite force upwards must be applied. This is usually exerted by the ground on all stationary bodies and gives rise to the idea of weight. The weight of a body is simply the force of gravity acting on it so **Weight = M g.**

If a body moves in a vertical direction, the force required must include the force of gravity.
F = Ma + Mg = M(a + g) and acceleration ‘a’ is positive when upwards.

For a hovering aircraft a = 0 and the upwards force is equal to its weight. A body accelerating upwards must exert a force greater than its weight. A body accelerating downwards must exert a force less than the weight.

SELF ASSESSMENT EXERCISE No. 6

1. Calculate the force required from a rocket engine if it must accelerate the rocket upwards at 3 m/s^2. The mass is 5 000 kg.
2. A lift of mass 500 kg is accelerated upwards at 2 m/s^2. Calculate the force in the rope.
3. The same lift is accelerated downwards at 2 m/s^2. What is the force in the rope then?

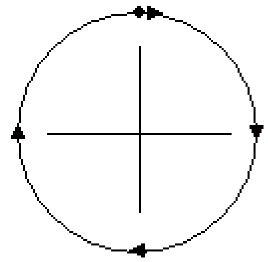
6. Angular Motion

This work applies to bodies revolving about a point at a fixed radius so the path taken by any point on the body is a circular path. This could apply to wheels of many forms, to vehicles (land, sea and air) going around a curve.

6.1. Angles

Angle has no units since it a ratio of arc length to radius. We use the names ***Revolution, Degree and Radian***.

Revolution
A point on a wheel that rotates one revolution traces out a circle. One revolution is the angle of rotation. This is a bit crude for use in calculations and we need smaller parts of the revolution.


1 revolution

Degrees
Traditionally we divide one revolution into 360 parts and call this a degree with symbol ^{o}.
1 revolution = 360^o

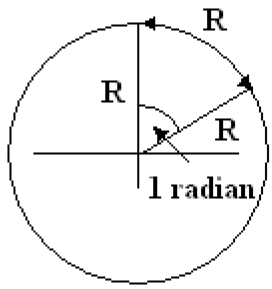
A single degree is not accurate enough for many applications so we divide a degree up into smaller parts called minutes. 1^o = 60 minutes or 60'

A minute can be divided up into even smaller bits called seconds and 1 minute = 60 seconds or 60".

In modern times we use decimals to express angles accurately so you are unlikely to use minutes and seconds.

Grads
In France, they divide the circle up into 400 parts and this is called a Grad. 1 revolution = 400 Grad. This makes a quarter of a circle 100 Grads whereas in degrees it would be 90^o.

Radian
In Engineering and Science, we use another measurement of angle called the Radian. This is defined as the angle created by placing a line of length 1 radius around the edge of the circle as shown. In mathematical words it is the angle subtended by an arc of length one radius. This angle is called the ***Radian***.



The circumference of a circle is 2πR. It follows that the number of radians that make a complete circle is:

$$\frac{2\pi R}{R} = 2\pi$$

There are 2π radians in one revolution so $360^o = 2\pi$ radian. 1 radian = 360/2π = 57.296^o

In the following work we will be using degrees and radian so it is very important that you make sure your calculator is set to the units that you are going to use. You might find a button labelled DRG on your calculator. Press this repeatedly until the display shows either D (for degrees) or R (for Radian) or if you are French, G (for Grad). On other calculators you might have to do this by using the mode button so read your instruction book. Also note that since one revolution is 2π radian and also 360^o we convert degrees into radian as follows.

$$\theta \text{ (radian)} = \text{degrees} \times 2\pi/360 = \text{degrees} \times \pi/180$$

6.2 Angular Velocity ω

The symbol for angular velocity is the lower case of the Greek letter Omega - ω.
If a body turns at constant speed, the angular velocity is the angle turned in 1 second. The angle turned in t seconds is then:

$$\theta = \omega t \text{ and so } \omega = \frac{\theta}{t} \text{rad}/s$$

If the body is speeding up or slowing down we may express the instantaneous angular velocity as the rate of change of angle per second.
In calculus form we can write:

$$\omega = \frac{d\theta}{dt}$$

In practical cases, angular velocity or speed is usually given in revolutions/second or revolutions/minute. When solving problems we nearly always have to convert this into radians/s. Since a circle (or revolution) is 2π radian we convert rev/s into rad/s by $\omega = 2\pi N$.

WORKED EXAMPLE No. 8

A wheel rotates 200^o in 4 seconds. Calculate the following.

i. The angle turned in radians?
ii. The angular velocity in rad/s

SOLUTION

$\theta = (200/180)\pi = 3.49$ rad. $\omega = 3.49/4 = 0.873$ rad/s

SELF ASSESSMENT EXERCISE No. 7

1. A wheel rotates 5 revolutions in 8 seconds. Calculate the angular velocity in rev/s and rad/s. (Answers 0.625 rev/s and 3.927 rad/s)

2. A disc spins at 3 000 rev/min. Calculate its angular velocity in rad/s. How many radians has it rotated after 2.5 seconds? (Answers 314.2 rad/s and 785.4 rad)

6.3 Angular Acceleration α

Angular acceleration (symbol α - alpha) occurs when a wheel speeds up or slows down. It is defined as the rate of change of angular velocity. Since angular velocity is the rate of change of angle we can also say that acceleration is the rate of change of the rate of change of angle.

If the wheel changes its velocity by Δω in t seconds, the acceleration is $\alpha = \Delta\omega / t$ rad/s^2

In calculus form we can express this as:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\omega}{dt^2}$$

WORKED EXAMPLE No. 9

A disc is spinning at 2 rad/s and it is uniformly accelerated to 6 rad/s in 3 seconds. Calculate the angular acceleration.

SOLUTION

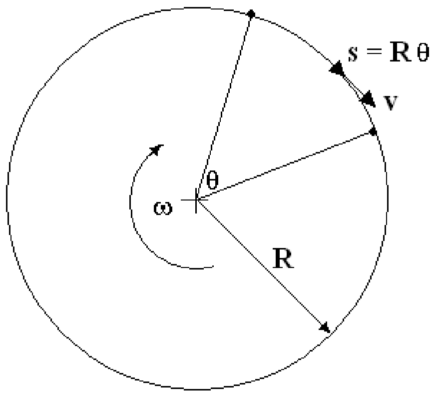
$\alpha = \Delta\omega/t = (\omega_2 - \omega_1)/t = (6 - 2)/3 = 1.33$ rad/s^2

EXERCISE No. 8

1. A wheel at rest accelerates to 8 rad/s in 2 seconds. Calculate the acceleration. (Answer 4 rad/s^2)

2. A flywheel spins at 5 000 rev/min and is decelerated uniformly to 2 000 rev/min in 12 seconds. Calculate the acceleration in rad/s^2. (Answer -26.2 rad/s^2)

6.4 Link between Angular and Linear Motion



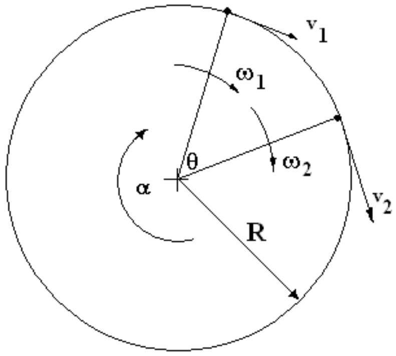
Consider a point moving on a circular path as shown.

In time t it rotates about the centre by angle θ and travels along the arc. The distance travelled on the circular path is the length of the arc 's' and:

$$\mathbf{s = R\theta}$$

The velocity along the circular path is **v = s/t = Rθ/t**

$$\mathbf{v = R\,\omega}$$



Next suppose that the point accelerates from angular velocity ω_1 to ω_2. The velocity along the curve also changes from v_1 to v_2.

Angular acceleration = $\alpha = (\omega_2 - \omega_1)/t$

Substituting $\omega = v/R$

$$\alpha = \frac{\left(\frac{v_2}{R} - \frac{v_1}{R}\right)}{t} = \frac{a}{R}$$

hence:

$$\mathbf{a = R\,\alpha}$$

It is apparent that to change an angular quantity into a linear quantity all we have to do is multiply it by the radius.

WORKED EXAMPLE No. 10

A car travels around a circular track of radius 40 m at a velocity of 8 m/s. Calculate its angular velocity.

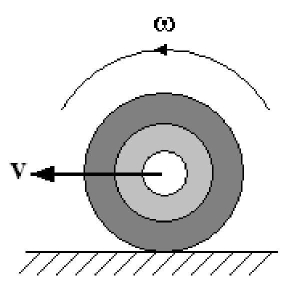
SOLUTION

$v = \omega R$ $\quad \omega = v/R = 8/40 = 0.2$ rad/s

WORKED EXAMPLE No. 11

A car has wheels 1 m diameter. Calculate the angular speed of the wheels in rev/minute when the vehicle travels at 100 km/h.

SOLUTION



If the vehicle moves forwards at v m/s then the surface of the wheel must be moving at v m/s otherwise it would be sliding on the surface.

v = 100 000 m/h or 100 000/3600 m/s

v = 27.777 m/s

$\omega R = \omega \times 0.5$ $\quad \omega = 27.777/0.5 = 555.555$ rad/s

$N = \omega/2\pi = 555.555/2\pi = 8.842$ rev/s or 530.5 rev/minute

SELF ASSESSMENT EXERCISE No. 9

1. The vanes on a steam turbine rotate at 3 000 rev/min at a mean radius of 0.3 m. What is the linear velocity of the vanes?

(Answer 94.2 m/s)

2. A winch rotates at 30 rev/min. The mean radius to the centre of the rope being drawn in is 0.2 m. Calculate the speed of the rope in m/s.

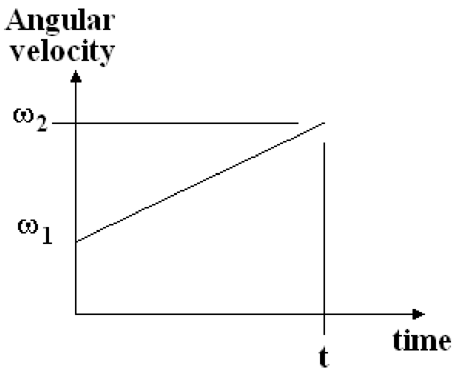
(Answer 0.628 m/s)

3. The wheels on a train are 1.5 m diameter and they revolve at 25 rev/min without slipping. What is the speed of the train in km/h?

(Answer 7.0686 km/h)

6.5 *Equations of Motions*

If a point moving at angular velocity ω_1 accelerates uniformly to ω_2 at a rate of α rad/s^2 in time t seconds, the graph is as shown. By the same reasoning as used for linear motion:



Angle rotated

$$\boldsymbol{\theta = \omega_1 t + \frac{(\omega_2 - \omega_1)t}{2} \ldots\ldots\ldots (1)}$$

Multiply out

$$\theta = \omega_1 t + \frac{\omega_2 t}{2} - \frac{\omega_1 t}{2} = \frac{\omega_2 t}{2} + \frac{\omega_1 t}{2} = \frac{t}{2}(\omega_2 + \omega_1)$$

The acceleration = ω

$$\alpha = \frac{(\omega_2 - \omega_1)}{t} \text{ hence } (\omega_2 - \omega_1) = \alpha t$$

Substituting this into equation (1) gives:

$$\boldsymbol{\theta = \omega_1 t + \frac{\alpha t^2}{2} \ldots\ldots\ldots (2)}$$

Since $\omega_2 = \omega_1$ + the increase in velocity $\omega_2 = \omega_1 + \alpha t$

Squaring we get:

$$\boldsymbol{\omega_2{}^2 = \omega_1{}^2 + 2\alpha\left[\frac{\alpha t^2}{2} + \omega_1 t\right] = \omega_1{}^2 + 2\alpha\theta \ldots\ldots\ldots (3)}$$

If we compare these with the equations for linear motion we see they are the same.

LINEAR	ANGULAR
$a = \frac{(v - u)}{t}$	$\alpha = \frac{(\omega_2 - \omega_1)}{t}$
$s = ut + \frac{at^2}{2}$	$\theta = \omega_1 t + \frac{\alpha t^2}{2}$
$s = \frac{(u - v)t}{2}$	$\theta = \frac{(\omega_2 + \omega_1)t}{2}$
$v^2 = u^2 + 2as$	$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$

SELF ASSESSMENT EXERCISE No. 10

1. A wheel accelerates from rest to 3 rad/s in 5 seconds. Sketch the velocity - time graph and determine the angle rotated.

(Answer 7.5 radian).

2. A wheel accelerates from rest to 4 rad/s in 4 seconds. It then rotates at a constant speed for 3 seconds and then decelerates uniformly to rest in 5 seconds. Sketch the velocity time graph and determine

   i. The angle rotated. (30 radian)
   ii. The initial angular acceleration. (1 rad/s^2)
   iii. The average angular velocity. (2.5 rad/s)

3. A roller 0.4 m diameter rolls down a slope starting from rest. It takes 10 seconds make 6 complete rotations along the sloping surface accelerating uniformly as it moves. Calculate the following:

   i. The angular velocity at the end. (7.54 rad/s)
   ii. The linear velocity at the end. (1.508 m/s)
   iii. The angular acceleration. (0.754 rad/s^2)
   iv. The linear acceleration. (0.151 m/s^2)
   v. The distance travelled. (7.54 m)

4. A large centrifugal air compressor rotates at 360 rev/min. The power is turned off and it decelerates to rest in 5 seconds. What is the angular acceleration?

(-7.54 rad/s^2)

5. A large steam turbine rotating at 3000 rev/min has the steam supply cut off and it slows down at a uniform rate to 500 rev/min in 50 seconds. What is the angular acceleration?

(-5.235 rad/s^2)

6. Assuming the turbine in Q5 continues to decelerate at the same rate, how long does it take to come to rest from the time the steam was cut off?

(10 s)