SOLID MECHANICS

BALANCING

TUTORIAL 1 – BALANCING OF ROTATING BODIES

On completion of this tutorial you should be able to do the following.

- Explain the importance of balancing.
- Explain static and dynamic balance.
- Solve problems involving coplanar balance.
- Solve problems involving balancing in four planes.
- Construct vector diagrams.
- Use vector resolution to solve problems.
- Extend the work to more complex problems.

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1. ***Introduction***

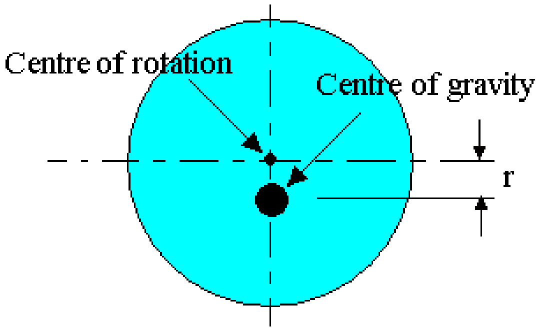
The balancing of rotating bodies is important to avoid vibrations. In heavy industrial machines such as steam turbines and electric generators, vibration could cause catastrophic failure. Vibrations are noisy and uncomfortable and when a car wheel is out of balance, the ride is quite unpleasant. In the case of a simple wheel, balancing simply involves moving the centre of gravity to the centre of rotation but as we shall see, for longer and more complex bodies, there is more to it. For a body to be completely balanced it must have two things.

1. ***Static Balance.*** This occurs when there is no resultant centrifugal force and the centre of gravity is on the axis of rotation.

2. ***Dynamic Balance.*** This occurs when there is no resulting turning moment along the axis.

2. Balancing In One Plane

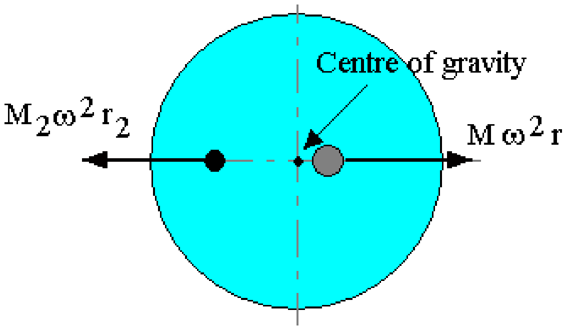
If the system is a simple disc then static balance is all that is needed. Consider a thin disc or wheel on which the centre of gravity is not the same as the centre of rotation. A simple test for static balance is to place the wheel in frictionless bearings. The centre of gravity will always come to rest below the centre of rotation (like a pendulum). If it is balanced it will remain stationary no matter which position it is turned to.



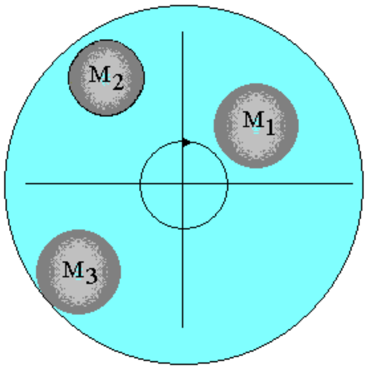
If the centre of gravity is distance r from the centre of rotation then when it spins at ω rad/s, centrifugal force is produced. This has a formula C.F.= $M\omega^2 r$ where M is the mass of the disc. This is the out of balance force. In order to cancel it out an equal and opposite force is needed. This is simply done by adding a mass M_2 at a radius r_2 as shown. The two forces must have the same magnitudes.

$$M\omega^2 r = M_2\omega^2 r_2$$
$$M r = M_2 r_2$$

Placing a suitable mass at a suitable radius moves the centre of gravity to the centre of rotation. This balance holds true at all speeds down to zero hence it is balanced so long as the products of M and r are equal and opposite.



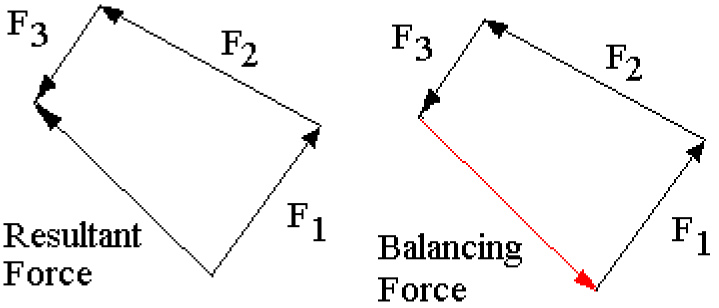
Now consider that our disc is out of balance because there are three masses attached to it as shown. The 3 masses are said to be coplanar and they rotate about a common centre.



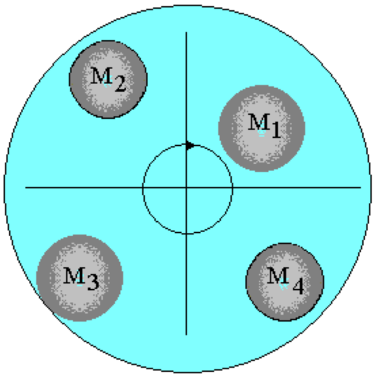
The centrifugal force acting on each mass is $F = M\,r\,\omega^2$. The radius of rotation is r and the angular velocity is ω in radians/second. The force acting on each one is hence

$F_1=M_1r_1\omega^2$ $F_2=M_2r_2\omega^2$ $F_3=M_3r_3\omega^2$

These are vector quantities and we can add them up to find the resultant force as shown.

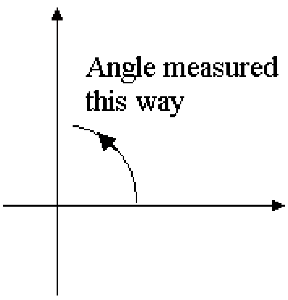


If the system was balanced, there would be no resultant force so the force needed to balance the system must be equal and opposite of the resultant (the vector that closes the polygon). The balancing mass M_4 is then added at a suitable radius and angle such that the product M r is correct.



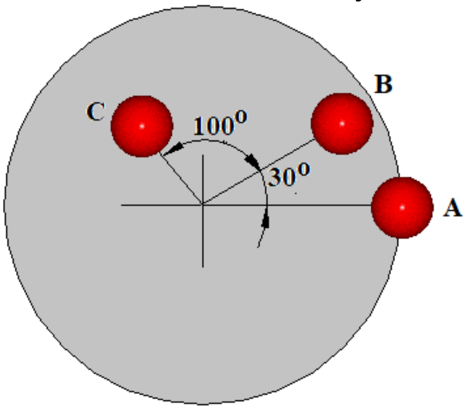
The result obtained would be the same whatever the value of ω and when ω = 0 we have static balance. In order to make the solution easier, we may make ω = 1 and calculate M r for each vector. This is called the M r polygon or vector diagram.

Note that angles will be given in normal mathematical terms with anticlockwise being positive from the x axis as shown.



WORKED EXAMPLE No. 1

Three masses A, B and C are placed on a balanced disc as shown at radii of 120 mm, 100 mm and 80 mm respectively. The masses are 1 kg, 0.5 kg and 0.7 kg respectively. Find the 4th mass which should be added at a radius of 60 mm in order to statically balance the system.

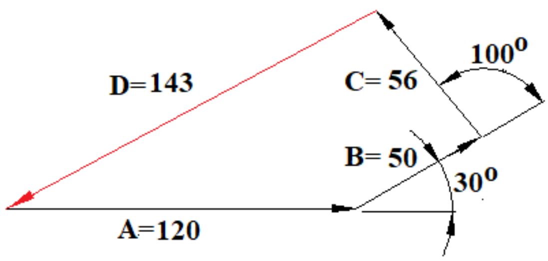


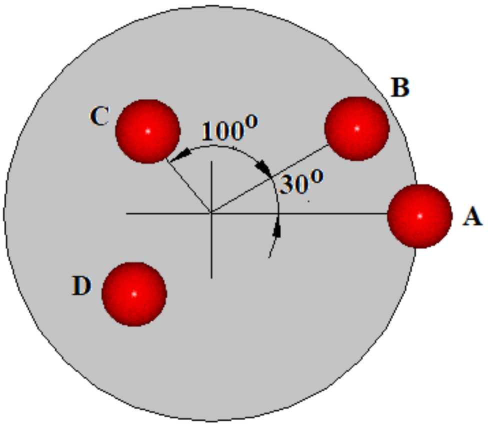
SOLUTION

First draw up a table to calculate the value of M r for each mass.

	Mass	radius	M r
A	1	120	120
B	0.5	100	50
C	0.7	80	56
D	M_D	60	60 M_D

Draw the M r polygon to find the value of M r for the 4th mass.



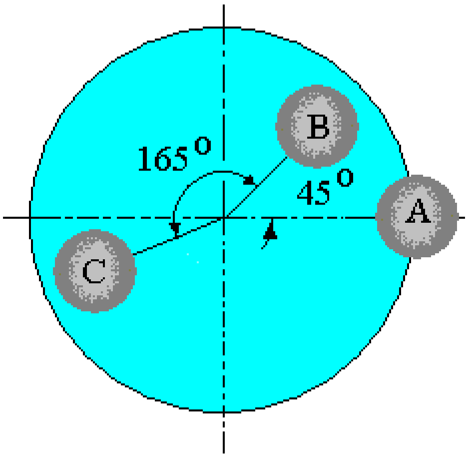


The resultant is 144.3 kg mm and is equal to 60 M_D.
The mass required is 144.3/60 = 2.4 kg 208° anticlockwise of A as shown.

SELF ASSESSMENT EXERCISE No. 1

Find the 4th mass that should be added at a radius of 50 mm in order to statically balance the system shown.

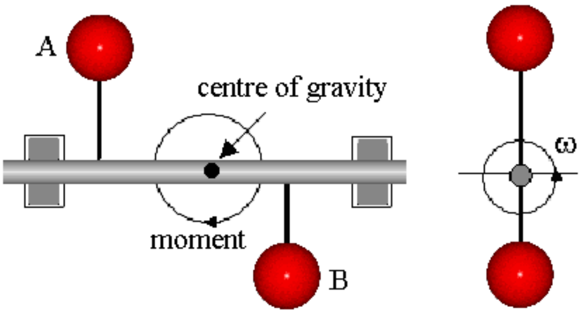
Mass A is 1 kg at 100 mm radius.
Mass B is 1.5 kg at 75 mm radius
Mass C is 2.0 kg at 90 mm radius.



Answer 0.52 kg at 156° clockwise from A.

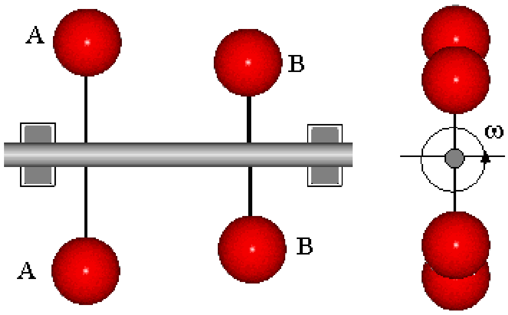
3. Masses Not In the Same Plane

Consider 2 masses statically balanced as shown but acting at different places along the axis.

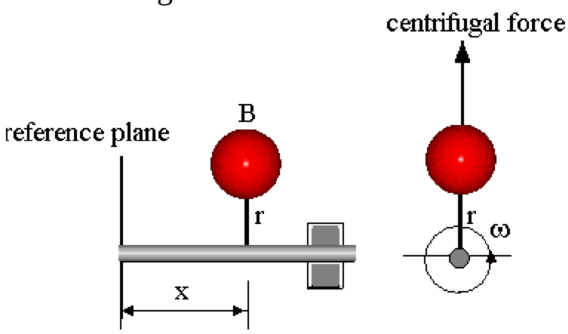


For static balance $M_A r_A = M_B r_B$

It is clear that even with static balance, centrifugal force will produce a turning moment about the centre of gravity for the system. In this simple case, the problem is solved by adding equal and opposite forces at the two points as shown.



Consider the turning moment due to a single mass.



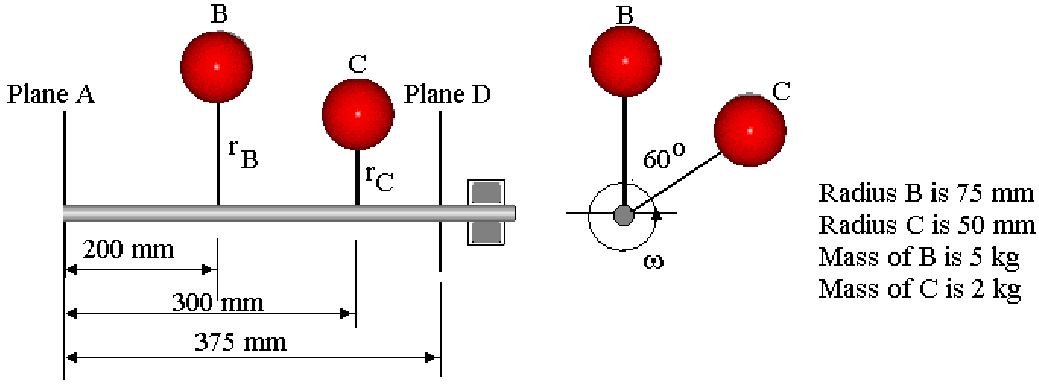
The centrifugal force produced is $F = Mr\omega^2$

The turning moment about the reference plane = T.M. = F x = $Mr\omega^2 x$

For dynamic and static balance we must work out the resultant turning moment and add masses at appropriate points to cancel it out. The appropriate points will be on two planes not coplanar with any of the original masses. This involves drawing two vector diagrams and since ω is common to all vectors we can again take ω =1 and draw vectors representing Mr and Mrx. This is best explained with a worked example.

WORKED EXAMPLE No. 2

Find the mass and the angle at which it should be positioned in planes A and D at a radius of 60 mm in order to produce complete balance of the system shown.



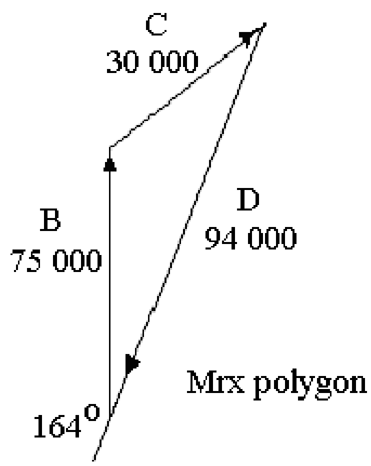
SOLUTION

Note that the diagram has been drawn with B vertical. It is a good idea to always start by making one of the known masses horizontal or vertical to make the construction of the vector diagrams easier. All angles should be expressed in absolute terms.

Plane A is the reference plane. All values of x are measured from plane A thus making Mrx for A equal to zero. It follows that it does not appear in the vector diagram. Make up a table as follows leaving unknowns as symbols.

	M	r	Mr	x	Mrx
A	M_A	60	$60M_A$	0	0
B	5	75	375	200	75 000
C	2	50	100	300	30 000
D	M_D	60	$60M_D$	375	$22\,500M_D$

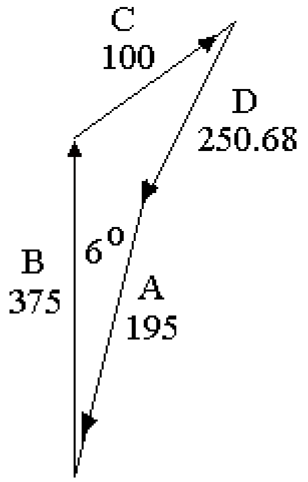
Now draw a polygon of Mrx vectors in order to find the value of Mrx at D. Start with B in this case because it is vertical.



Scaling the vector D which closes the triangle we find
Mrx for D = 94 000 = 22 500 M_D
Hence M_D = 94 000/22 500 = 4.17 kg and it is positioned 254o

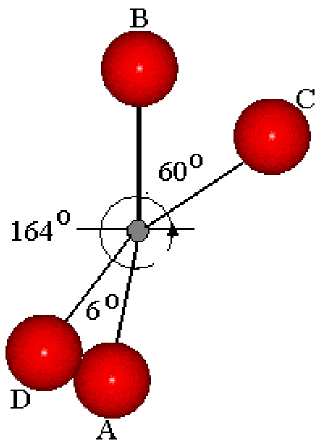
Now we calculate Mr for D.
Mr for D = 4.178 x 60 = 250.68.

Next we draw the polygon for the Mr values as shown.



The vector which closes the polygon represents Mr for point A
Mr for A is 195.
60M_A = 195 M_A = 195/60 = 3.25 kg at 6° to the vertical.

The answer is best shown with an end view.

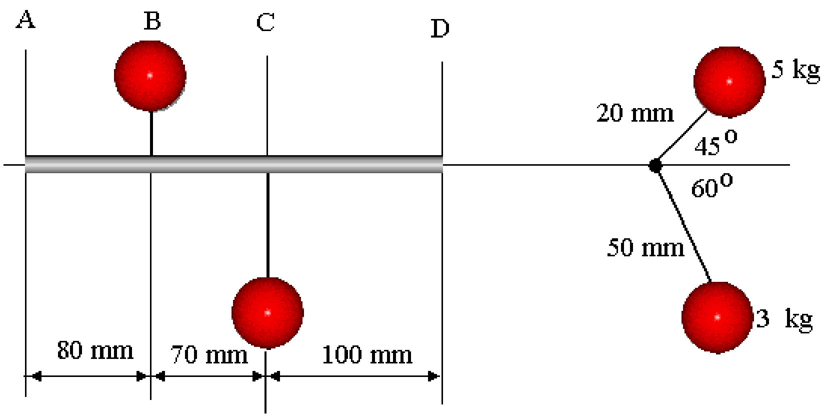


SELF ASSESSMENT EXERCISE No. 2

1. A shaft has 4 discs A, B, C and D along its length 100 mm apart. A mass of 0.8 kg is placed on B at a radius of 20 mm. A mass of 2 kg is placed on C at a radius of 30 mm and rotated 120^o from the mass on B. Finf the masses to be placed on A and D at a radius of 25 mm that wil produce total balance.

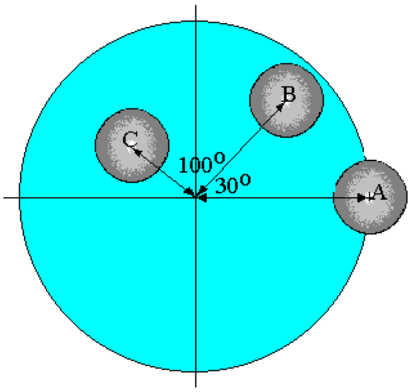
(Answers 0.696 kg and 1.52 kg)

2. The diagram below shows masses on two rotors in planes B and C. Determine the masses to be added on the rotors in planes A and D at radius 40 mm which will produce static and dynamic balance.(1.9 kg at 177^o and 2.2 kg at 141^o)

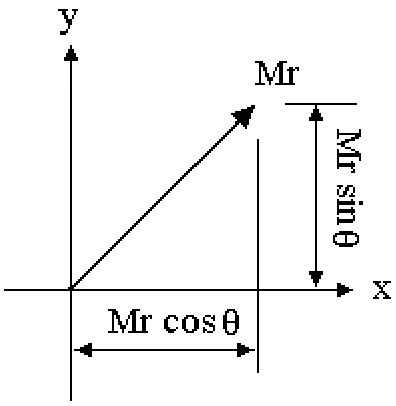


4 *Solution By Using Vector Resolution*

A more accurate approach to solving the vector diagrams in the preceding work is to resolve each vector into vertical and horizontal components. The resultant vector is then found by adding these components. Consider worked example No.1 again.



Each vector has a component in the x direction given by Mr cosθ and in the y direction it is MRsinθ



Work out these for each vector and include it in the table.

	Mass	radius	M r	θ	Mr cosθ	MRsinθ
A	1	120	120	0°	120	0
B	0.5	100	50	30°	43.3	25
C	0.7	80	56	130°	-36	42.9
			Totals		127.3	67.9

The resultant vector has x and y components of 127.3 and 67.9. This can be solved with Pythagoras. Resultant Mr = $(127.3^2 + 67.9^2)^{½}$ = 144.3 kg mm as before. The mass required is 144.3/60 = 2.4 kg. The angle ϕ = atan (67.9/127.3) or $\tan^{-1}$(67.9/127.3) = 28°

The balancing force is 180° anticlockwise of this so the balancing mass must be placed at an angle of 208°.

Here is the solution to worked example No. 2 using the same method.

	Mass	radius	M r	x	Mrx	θ
A	M_A	60	60 M_A	0	0	unknown
B	5	75	375	200	75 000	90
C	2	50	100	300	30 000	30
D	M_D	60	60 M_D	375	22 500 M_D	unknown

	MRx cosθ	MRxsinθ
A	0	0
B	0	75 000
C	25 980	15 000
D	-25 980	-90 000
Totals	0	0

If balanced, the totals must be zero so the unknown values may be deduced.

Resultant Mrx for D = $(25\,980^2 + 90\,000^2)^{½}$ = 93 670

This is the value for D so 22 500 M_D = 93 670 M_D =9 360/22 500 = 4.16 kg

$\phi = \tan^{-1}(90\,000/25\,980) = 73.98^o$ so the true angle θ is $180^o + 73.98^o = 253.8^o$

Now fill in the table for the Mr values. Again the totals must be zero so we can deduce the values for A.

	M	r	M r	θ	Mr cosθ	MRsinθ
A	M_A	60	60 M_A		-17.3	-240
B	5	75	375	90	0	375
C	2	50	100	30	86.6	50
D	4.167	60	250	253.8^o	-69.3	-240
				Totals	0	0

Resultant Mr vector is $(17.3^2 + 185^2)^{½}$ = 185.8 and this is equal to 60 MA.

M_A = 185.8/60 = 3.1 kg

ϕ = = $\tan^{-1}(185/17.3) = 84.6^o$ so the true angle θ is $180^o + 84.6^o = 264.6^o$

This gives a more accurate answer than that produced by drawing and scaling.

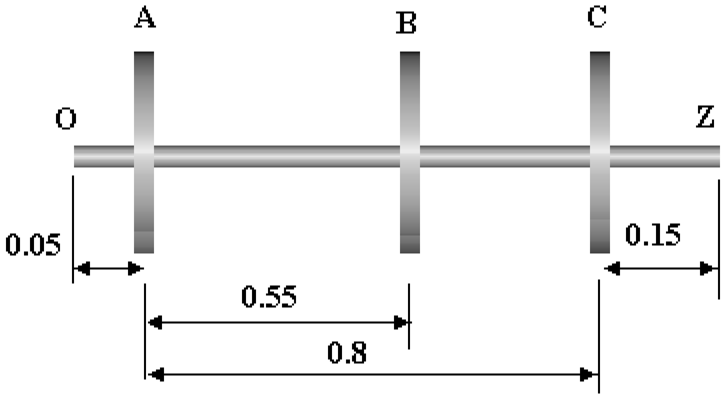
5. More Advanced Problems

Some other points to be considered are:

- The reference plane can be any of the existing planes.
- The moment about any point can be made zero by placing correction masses on one plane only if static balance is not important.

WORKED EXAMPLE No. 3

A rigid rotor caries three thin discs A, B and C mounted on a shaft 1 metre long running from O to Z. The distances from O are designated x in the table and the out of balance moment as MR in gramme metre. The out of balance force is at angle θ degrees.



TABLE

	x (m)	MR (g m)	θ (degrees)
A	0.05	20	0
B	0.6	10	120
C	0.85	20	-60

(a) Calculate correcting forces needed on planes A and C to bring the system into balance.

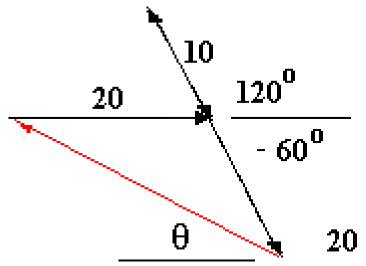
(b) The rotor is suspended from a bearing at O with the other end unrestrained. It can only be balanced on disc C. Calculate the corrective balance needed to ensure that end Z does not move. Calculate the magnitude of the force resulting at O when the shaft revolves at 500 rad/s.

SOLUTION

Make A the reference plane and re-tabulate as follows.

	x (m)	MR (g m)	θ (degrees)	MRx (g m^2)
A	0.0	20	0	0
B	0.55	10	120	5.5
C	0.8	20	-60	16

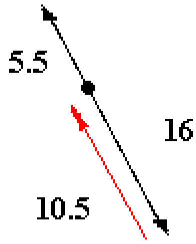
Construct the MR polygon.



The red vector produces balance and may be calculated or measured. The length is 26.5 g m and the angle θ is 19.1°. This is placed on plane A. The table may be further completed.

	x (m)	MR (g m)	θ (degrees)	MRx (g m^2)
A	0.0	20	0	0
B	0.55	10	120	5.5
C	0.8	20	-60	16
A2	0.0	26.5	160.9	0

Next construct the MRx polygon. This is simply two vectors at opposite angles giving a closing vector of 10.5 at 120°.



This is to be placed on C so the value of MR is 10.5/0.8 = 13.125 g m

This now produces an imbalance of 13.125 on the static balance (MR) so an equal and opposite force is added to A

The complete table is

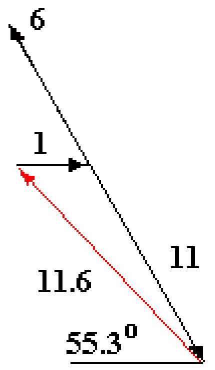
x (m)	MR (g m)	θ (degrees)	MRx (g m^2)	
A	0.0	20	0	0
B	0.55	10	120	5.5
C	0.8	20	-60	16
A2	0.0	26.5	-199.9	0
A3	0.0	13.125	-60	0
C2	0.8	13.125	120	10.5

A2 and A3 can be combined to a single MR value of 18.5 at 187 degrees. The final table is

(b) The key here is that the moment about end O must be zero to stop the other end moving and static balance must be ignored. Make O the reference plane and re-tabulate as follows with x measured from O

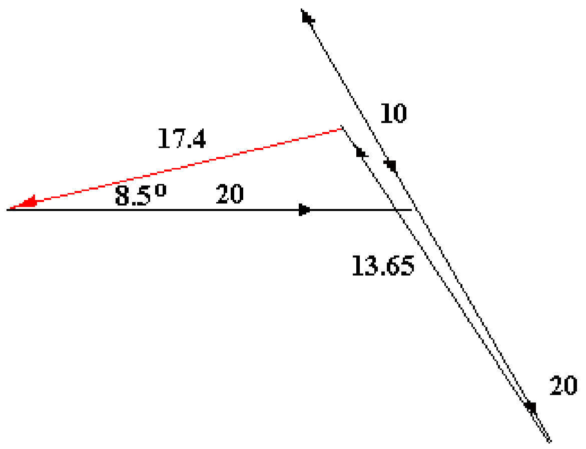
	x (m)	MR (g m)	θ (degrees)	MRx (g m^2)
A	0.05	20	0	1
B	0.6	10	120	6
C	0.85	20	-60	17

Construct the MR polygon. The closing vector is 11.6 at 124.7^o



This produces zero moment about end O and the balance needed on C is 11.6/0.85 = 13.65 g m at 124.7^o There is now a static imbalance that may be found from the MR polygon.

	x (m)	MR (g m)	θ (degrees)	MRx (g m^2)
A	0.05	20	0	1
B	0.6	10	120	6
C	0.85	20	-60	17
C2	0.85	13.65	124.7	11.6



A static imbalance of 17.4 g m at 188.5^o now exists and this will be countered by a force in the bearing at O. Centrifugal force = mω^2R so the force on the bearing is

$$MR \times \omega^2 = 17.4 \times 10^{-3} \text{ (kg m)} \times 500^2 = 4\ 350 \text{ N}$$

Note there are various methods of solving the vector diagrams such as drawing to scale or calculating the vertical and horizontal components of each vector and summing them to find the resultant.