MECHANICS OF SOLIDS
STATICS - PIN JOINTED FRAMES

This is set at the British Edexcel National level NQF 3.

On completion of this tutorial you should be able to explain and solve problems involving loaded pin jointed structures.

Contents

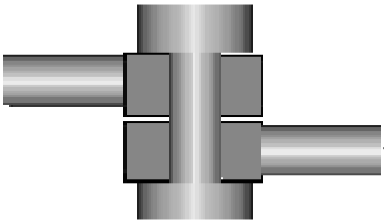
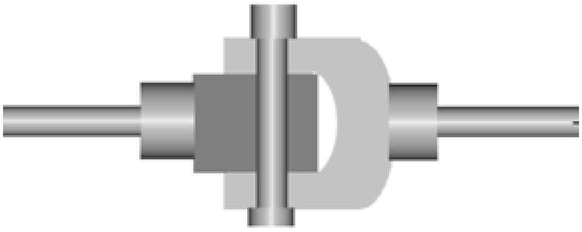
1. Introduction

A pin jointed frame is a structure of members joined together with pin joints. Although in reality such structures are bolted, riveted or welded, they are often treated as pin jointed. These structures are also called ***Latticed Frame Structures***. Examples are shown below.




2. Pin Joint

A true pin joint allows the joined members to swivel as opposed to a rigid joint that does not. A rigid joint may be welded but a pin joint may be a bolt, a rivet or any form of swivel pin.

The important points about a pin joint are:

- The connected members are free to rotate so they cannot exert a turning moment on each other.
- The force in the member can only pull or push along the line of the member.

3. Equilibrium

All static structures such beams and frames are in a state of equilibrium. This must mean that:

- All forces in any given direction add up to zero.
- All the turning moments about a given point must add up to zero.

If we only use Cartesian coordinates it follows that:

- All the vertical forces upwards (+ve) must equal all the vertical forces downwards (-ve). In other words $\mathbf{\Sigma F_x = 0}$
- All the horizontal forces to the right (+ve) must equal all the horizontal forces to the left (-ve) In other words $\mathbf{\Sigma F_y= 0}$
- All the clockwise turning moments (+ve) must equal all the anticlockwise turning moments (-ve). In other words $\mathbf{\Sigma M = 0}$

This is known as ***D'Alambert's Principle***.
It is of interest to note that in maths anti-clockwise is positive and the use of the opposite convention in some mechanical problems can cause confusion so bear this in mind.
Another important point is that if a body is in static equilibrium, all the force must be concurrent, i.e. they must all act through a common single point.

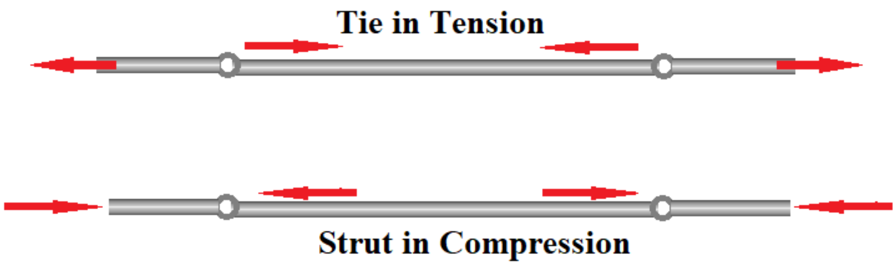
4. *Struts and Ties*

Consider a member (bar) with a pin joint at each end as shown below. A pin joint cannot transmit rotation (torque) from one to another so each can only push or pull on the joint along the direction of its length.

Remember also that the force in the other end of each member also pushes or pulls and so acts in the opposite direction with equal force.

A member in tension is called a ***Tie*** and is shown with arrows pointing inwards at each end because the force pulls on the joint.

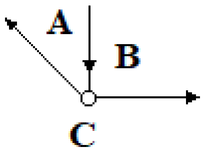
A member in compression is called a ***Strut*** and is shown with arrows pointing outwards at each end because the force pushes on the joint..



5. *Bow's Notation*

When three or more members are pinned together and static the resultant force must be zero. This means that if we add up all the forces as vectors, they must form a closed polygon. If one or even two of these forces is unknown, then it must be the vector, which closes the polygon. We need to be able to identify the members and to do this we use Bow's Notation.

Consider three members joined by a pin as shown in the figure.

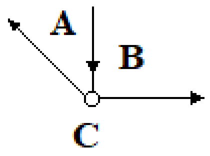


1. Label the spaces between each member. It doesn't matter what order you label them in. This is why the diagram is called a ***Space Diagram.***

2. Starting at any space, say A, identify each member by moving clockwise around the joint. In this case the first becomes a - b, the next b - c and the last c - a (in this case only).

SELF ASSESSMENT EXERCISE No. 1

Study the three members below. Identify each and write down which are ties and which are struts.



Member a - b is a ________________

Member b - c is a ________________

Member c - A is a ________________

Check your answers against the following.

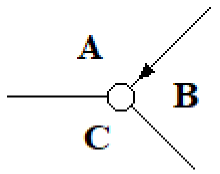
a - b pushes on the joint so it is in compression and is hence a strut.

b - c and c - a pull on the joint and so they are in tension and are hence ties.

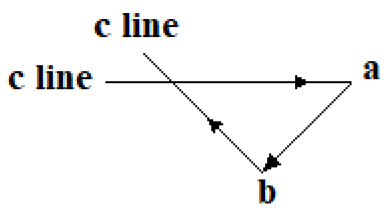
Now let's apply Bow's notation to a simple problem in order to solve the unknown forces. We draw a vector diagram to add them together and we know it must add up to zero. The vectors must be drawn at the correct angles as given or as measured.

Only one of these forces is known and this is important if we are to draw the vector diagram. Three forces will give us a ***Triangle of Forces***.

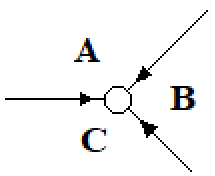
1. Identify the members as previously. The only known vector is a – b.



2. Draw the known vector a - b.
   We know that the next vector b - c starts at b but we do not know its length. Draw a 'c' line from 'b' in the direction of member b - c.
   We know that when all the vectors are added, they must form a closed triangle so c - a must end at 'a'.
   Draw a 'c' line through 'a' in the direction of member c - a. Where the two 'c' lines cross must be point 'c'.

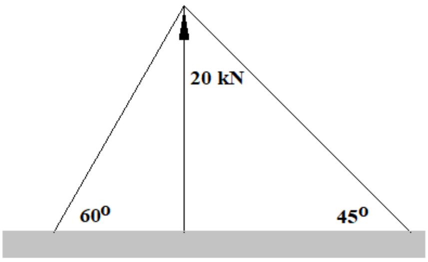


3. Finally, transfer the arrows back to the space diagram in the same direction as on the triangle of forces. If they push onto the pin joint, the member must be in compression and so is a strut. If the arrow pulls on the joint, the member must be in tension and so is a tie.



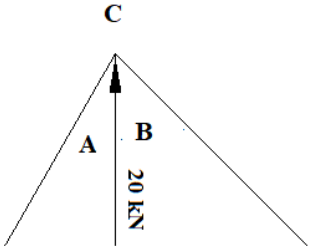
WORKED EXAMPLE No. 1

A strut is held vertical as shown by two guy ropes. The maximum allowable compressive force in the strut is 20 kN. Calculate the forces in each rope. Note that ropes can only be in tension and exert a pull. They cannot push.



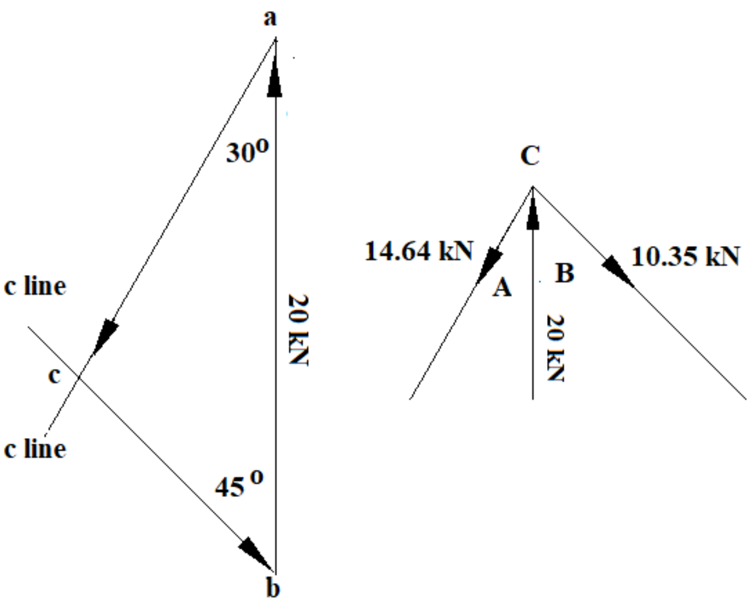
SOLUTION

First draw the space diagram.



Next draw the triangle of forces. Going clockwise around the joint the force is b – a 20 kN up. The next force is a – c so draw a c line through b.

The third and final force is c – b so draw a c line through b. hence finds point c. The arrows must end at the start point so the directions are as shown.



The forces in the ropes are b - c and c - a. These may be found by scaling or by trigonometry. Use the sin rule if you have learned it at this stage.

$$\frac{\text{bc}}{\sin 30^o} = \frac{20}{\sin 105^o} \qquad \text{bc} = 10.35 \text{ kN}$$

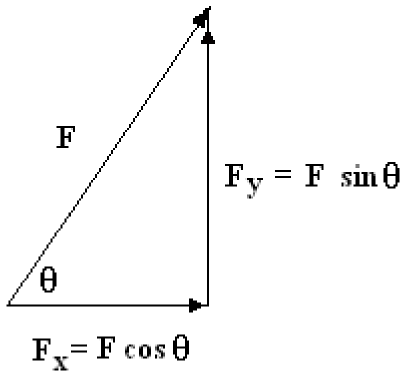
$$\frac{\text{ca}}{\sin 45^o} = \frac{20}{\sin 105^o} \qquad \text{ca} = 14.64 \text{ kN}$$

6. Resolution Method

This is an analytical way of solving the forces in pin jointed frames based on the condition of static equilibrium. An extension of this method is also called the Method of Sections but when used for uncomplicated examples the two are the same.

The force in any member may be resolved into a vertical component F_y and horizontal component F_x.

Consider a vector of magnitude F at angler θ as shown. Note that θ is measured anticlockwise from the positive x axis.



The vertical component of F is called F_y and may be found from trigonometry as $\mathbf{F_y = F \sin \theta}$
The horizontal component is called F_x and may be found from trigonometry as $\mathbf{F_x = F \cos \theta}$

We can also use Pythagoras' theorem to give

$$\mathbf{F} = \sqrt{\mathbf{F}_x^2 + \mathbf{F}_y^2}$$

If we add the vertical and horizontal components we get back to the original vector as shown.

In any static structure we know that $\mathbf{\Sigma F_x = 0}$ and $\mathbf{\Sigma F_x = 0}$ and $\mathbf{\Sigma M = 0}$

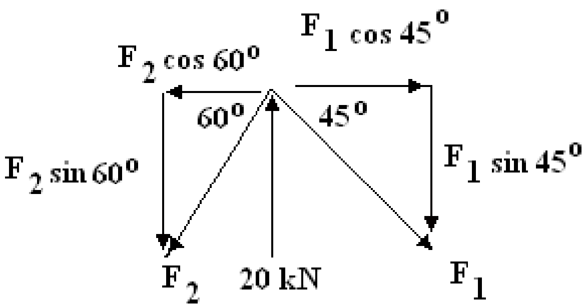
This must apply to any joint and being a pin joint, there can be no moment due to the force in the member.

WORKED EXAMPLE No. 2

Repeat example No. 1 using resolution of forces.

SOLUTION

Remember up is positive and right is positive.
Also note that the direction of F_1 and F_2 can only be pulling on the joint.



$\Sigma F_x = 0 = F_1 \cos 45^o - F_2 \cos 60^o$
$F_1 \cos 45^o = F_2 \cos 60^o$

$0.707 F_1 = 0.5 F_2$
$F_2 = 1.414 F_1$

$\Sigma F_y = 0 = 20 - F_1 \sin 45^o - F_2 \sin 60^o$
$\Sigma F_y = 0 = 20 - 0.707F_1 - 0.866F_2$

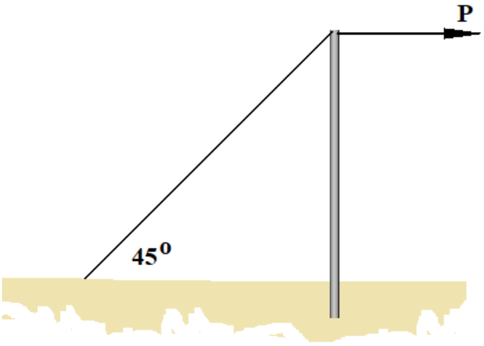
$0 = 20 - 0.707F_1 - 0.866(1.414 F_1)$
$0 = 20 - 0.707F_1 - 1.224F_1$
$0 = 20 - 1.931F_1$

$F_1 = 20/1.931 = 10.35$ N $\qquad$ $F_2 = 14.64$ kN

SELF ASSESSMENT EXERCISE No. 2

1. The strut is held upright by a rope and a horizontal force P acts on the top as shown.

   The maximum compressive force allowed in the strut is 5 kN. Determine by any method the maximum allowable force P and the corresponding force in the rope.



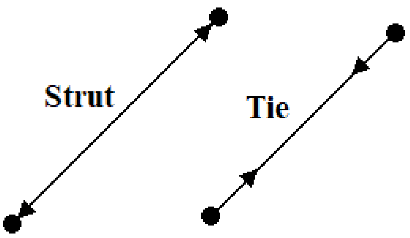
(Ans. 7.07 kN in rope and P= 5 kN)

7. *Solving the Forces in Pin Jointed Frames*

Let's now apply our knowledge to unknown forces in latticework frames.

Many of these structures are riveted and not entirely free to rotate at the joint but the theory of pin jointed frames seems to work quite well for them. We will apply Bow's notation to each joint in turn and so solve the forces in each member. By transferring the direction back from the polygon to the framework diagram, it can be deduced which are struts and which are ties. Knowing this, the force direction is determined at the other end of the member and this is needed to solve the other pin joints.

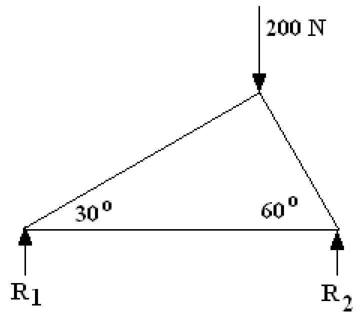
You learned earlier that a strut is a member in compression and a tie is a member in tension. They are drawn with the internal forces shown as follows.



The following worked example show you how solve a basic problem.

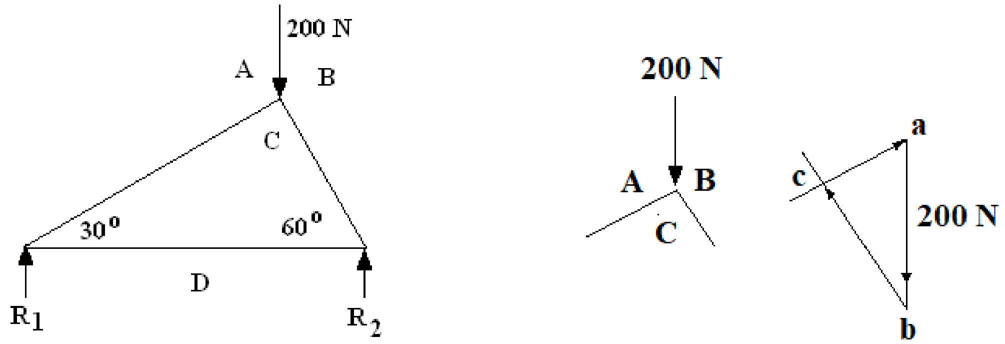
WORKED EXAMPLE No. 3

Solve the forces and the reactions for the frame shown.



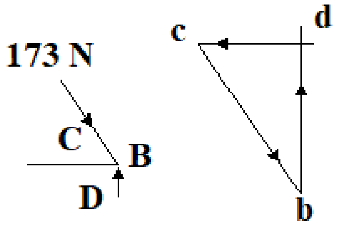
SOLUTION

First draw the space diagram and label the spaces. Next solve the joint with the known force.



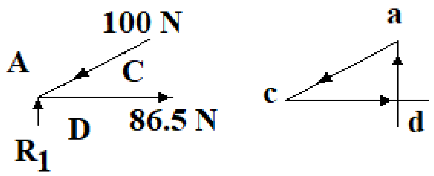
By scaling or use of trigonometry b - c = 173 N (strut) c - a = 100 N (strut)

Next solve the other joint.



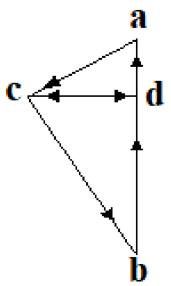
By scaling or trigonometry b - d = R_2 = 150 N. c - d = 86.5 N (Tie)

R_1 may easily be deduced since the total upwards force is 200 N.
R_1 must be 200 - 150 = 50 N. The solution for the other joint is not really needed but here it is.



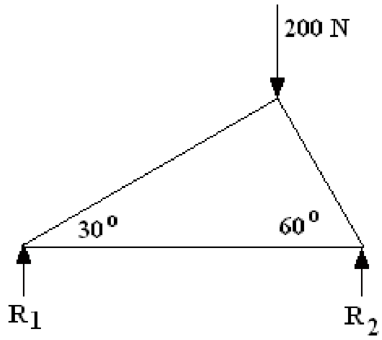
From this a - d = R_1 = 50 N as expected.

When you are proficient at this work, you may find it convenient to draw all the solutions together as one diagram. For this example we would have the figure shown.



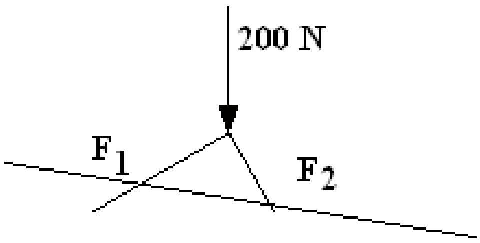
WORKED EXAMPLE No. 4

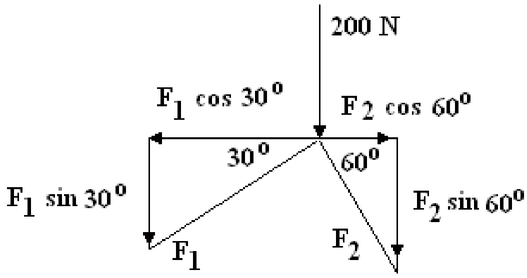
Repeat example 4 using the method of sections.



SOLUTION

We must start at a solvable joint and this is at the top. We draw a line to cut the section and balance the force on the section.





We don't know the directions of forces F_1 and F_2 so we must assume they are positive and let the solution tell us. Assume the forces are pulling on the joint as shown. Note we could use the previous (Bow's) notation $F_1 = F_{bc}$ and $F_2 = F_{ca}$ but it is for you to decide which you prefer.

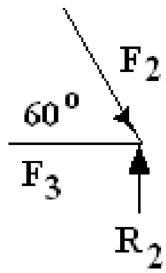
$\Sigma F_x = 0 = -F_1 \cos 30^o + F_2 \cos 60^o$ $\quad$ $0.866F_1 = 0.5 F_2$ $\quad$ $F_1 = 0.5774 F_2$

$\Sigma F_y = 0 = -200 - F_1 \sin 30^o - F_2 \sin 60^o$

$0 = -200 - 0.5F_1 - 0.866F_2$
$0 = -200 - 0.5(0.5774F_2) - 0.866F_2$
$0 = -200 - 0.2887F_2 - 0.866F_2$
$0 = -200 - 1.1547F_2$

$F_2 = -173.2$ N opposite to that assumed so pushing and hence a strut.
$F_1 = -100$ N opposite to that assumed so pushing and hence a strut.

Now solve one of the corners, say the right one. Note F_2 pushes.



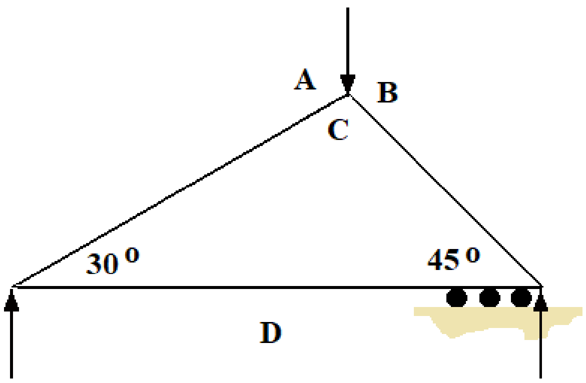
Vertical Balance gives $\quad$ $F_2 \sin 60^o + R_2 = 0$
$-173.2\,(0.866) + R_2 = 0$
$R_2 = 150$ N (up)

Horizontal Balance gives $\quad$ $F_2 \cos 60^o + F_3 = 0$
$173.2(0.5) + F_3 = 0$
$F_3 = -86.6$ N (to left pulling so a tie)

SELF ASSESSMENT EXERCISE No. 3

Solve the forces in the frameworks below and determine which ones are ties and which are struts.

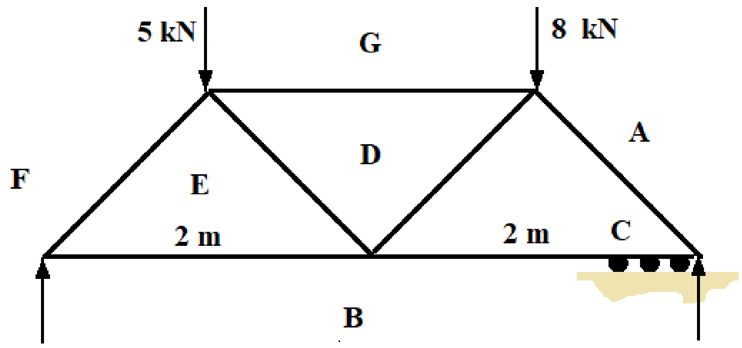
1.



Note that the rollers ensure R_2 is vertical.

(Ans. b - c 450 N strut, c - a 370 N strut, d - c 317 N tie, R_1=183 N R_2= 317 N)

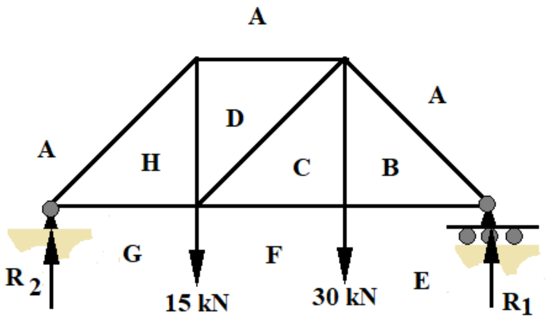
2.



All internal angles are 45°

Answers
R_1 = 5.75 kN. R_2 =7.25 kN
b - c 7.25 kN tie
a - c 10.25 kN strut
d - g 6.5 kN strut
c - d 1 kN strut.
e - f 8.2 kN strut
d - e 1 kN tie
e - b 5.7 kN. tie

3. The diagram below shows a plane pin-jointed framework subjected to vertical loads of 15 kN. and 30 kN. The frame is supported by a pin joint at the left-hand end and by rollers at the right hand end. Solve all the forces and reactions in the frame. The lengths of the vertical and horizontal members are 3 m. (note the spaces are not normally labelled for you).



Answers

R_1 = 25 kN. R_2 = 20 kN.

b - a = 35.5 kN. (strut)
e - b = 25 kN. (tie)
h - g = 20 kN. (tie)
h - a = 29 kN. (strut)
f - c = 25 kN. (tie)
c - d = 7.07 kN. (strut)
d - a = 20 kN. (strut)
h - d = 20 kN. (tie)
c - b = 30 kN. (tie)

Tip for solution. Start by finding the reactions and then solving one of the bottom corners where only 3 forces act.