MECHANICS OF SOLIDS

FATIGUE

You should judge your progress by completing the self-assessment exercises.

On completion of this tutorial you should be able to do the following.

- Explain fatigue.
- Explain the way fatigue affects different materials.
- Explain an S – N graph.
- Define fatigue strength.
- Define endurance limit.
- Explain the various factors that affect the fatigue life of a component.
- Predict the fatigue life of components.
- Predict the safety fatigue factor.

You should be familiar with basic stress and strain theory.

INTRODUCTION

Fatigue is a phenomenon that occurs in a material that is subject to a cyclic stress. Although the peak stress in each cycle is less than that needed to make the material fail in a tensile test, the material fails suddenly and catastrophically after a certain number of cycles.

Here are some examples of things that are subject to cyclic stress.

- ***Railway lines that bend every time a wheel passes over it.***
- ***Gear Teeth.***
- ***Springs.***
- ***The suspension cable on a suspension bridge every time a vehicle passes over it.***
- ***The skin and structural members of an aeroplane every time it flies.***
- ***A shaft with a pulley belt drive.***
- ***The connecting rod in a reciprocating engine.***
- ***The stub axle on a vehicle wheel.***

STRESS FLUCTUATION

Consider the case of an electric motor with a pulley drive on its shaft.

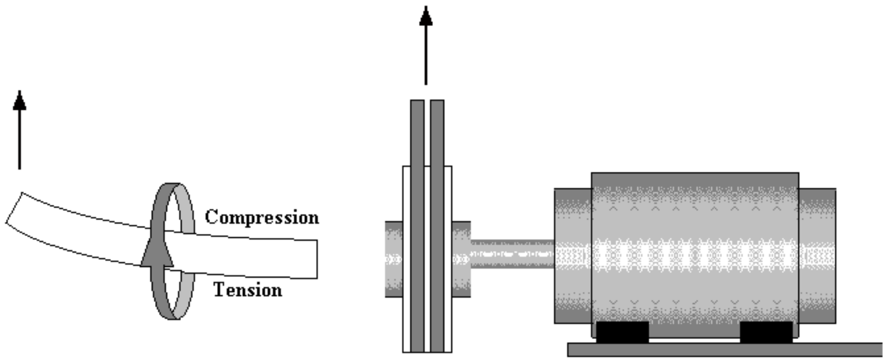


Figure 1

The shaft bends as shown producing tension on one side and compression on the other. As the shaft rotates, any given point on the surface experiences a direct stress that changes from tension to compression once every revolution. The alternating or fluctuating stress causes the failure. A stress - time graph is likely to be sinusoidal in a case like this.

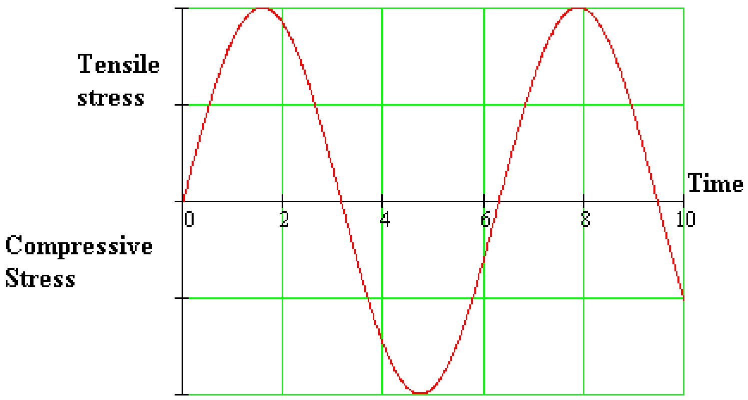


Figure 2

The fatigue life of a component depends on the values of the fluctuation, the mean stress level and the way the stress varies with time.

For example, on a bridge, the changes might be erratic and sudden from one value to another value without ever going into compression. It might be like this.

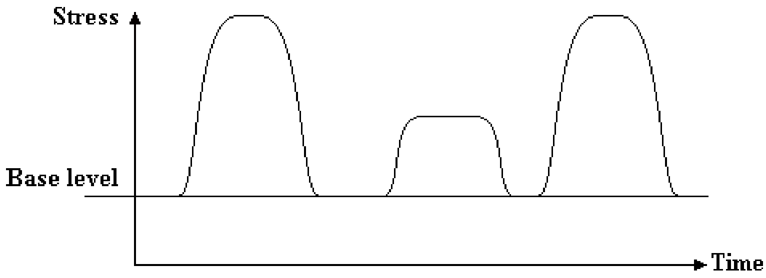


Figure 3

Steel components never fail in fatigue if the maximum stress level never exceeds a critical value called the endurance limit. If fatigue failure is possible, we need to predict how many cycles of stress are likely to cause failure.

FATIGUE TESTING

A fatigue test should ideally reproduce the same stress levels and fluctuations as in service. The most common form of test is the Wohler Test. In this test, the specimen is held in a chuck with a weight pan suspended from the end as shown. Each revolution bends the specimen so that the surface stress fluctuates between equal tensile and compressive values with a mean level of zero. The maximum stress is easily calculated. The test is repeated with different weights and hence different stress levels. It is rotated until it fails and the number of revolutions is counted. This is the number of stress cycles to failure.

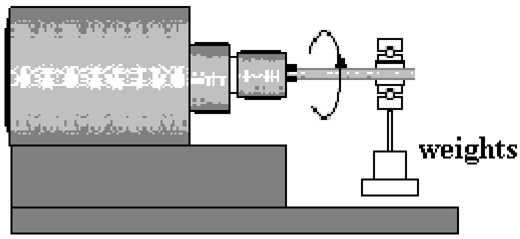


Figure 4

S – N GRAPHS

Test data is presented on a S - N graph. S stands for stress and N for the number of cycles. The symbol used for stress is σ (sigma). The graph is normally plotted with logarithmic scales as shown. This tends to straighten out the graphs. The diagram shows a typical result for steel.

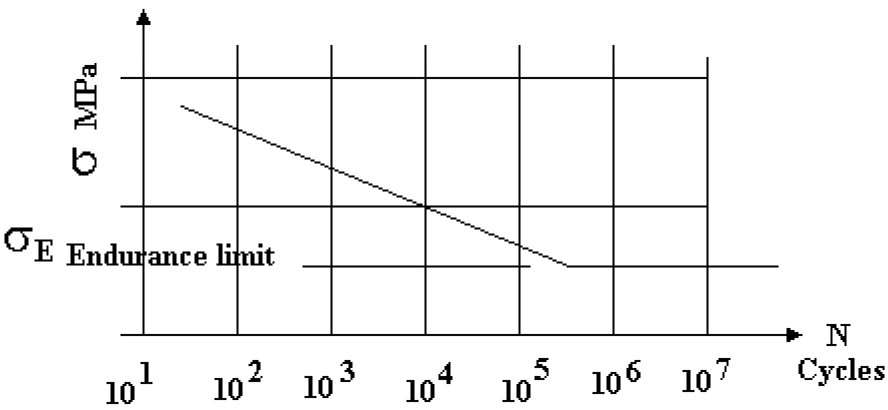


Figure 5

FATIGUE STRENGTH

The fatigue strength is the stress level that produces failure after a specified number of cycles.

ENDURANCE LIMIT

The lower limit σ_E is called the endurance limit. If the stress level is below this limit, it will never fail. Non-ferrous materials have no endurance limit. The diagram shows approximate fatigue characteristics of three materials. Research shows that for ferrous materials the endurance limit is approximately proportional to the tensile strength σ_u. A conservative relationship is $\sigma_E = 0.3\sigma_u$

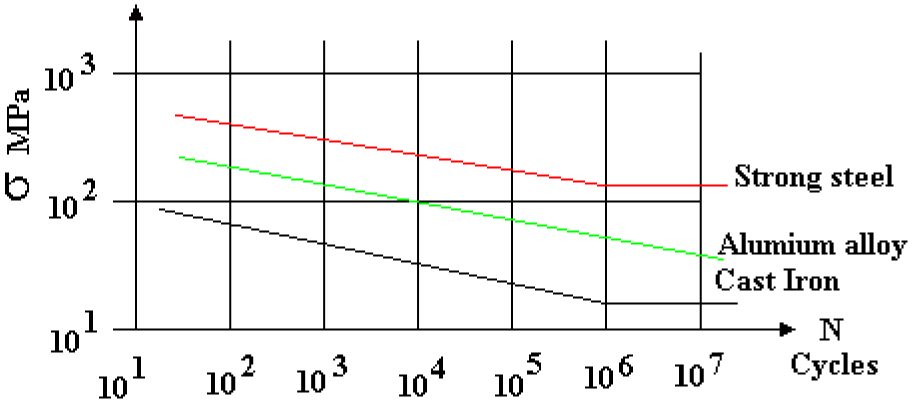


Figure 6

For materials with no clear endurance limit, σ_N values are stated instead. This is the number of cycles required to produce failure at the specified stress amplitude.

WORKED EXAMPLE No.1

Determine the fatigue strength of a strong steel that gives a life of 10 000 cycles. Use the graph above. What is the endurance limit?

SOLUTION

From the graph, the stress corresponding to 10 000 (10^4) on the red graph is approximately 300MPa. The endurance limit is approximately 200 MPa.

CRACK FORMATION

The crack usually starts at some surface defect or feature that produces a stress concentration. For example, an undercut in a shaft for a circlip or a hole for a pin would cause stress concentration. Any fault in the material such as a slag inclusion will also initiate a crack. Undercuts should have rounded corners to reduce this to a minimum. If the material is ductile, the initial crack will not spread easily and the crack opens up and closes as the stress fluctuates. This wears the surface of the crack smooth. As the crack progresses, new material is exposed which starts to wear smooth.

Figure 7

When the crack has spread enough to reduce the cross sectional area of the material to a point where it can no longer carry the load, sudden failure occurs. Often the fracture has an OYSTER SHELL appearance due to the early stages being worn smoother than the later stages.

Cracks spread more easily in brittle material, especially at cold temperatures and failure is sudden.

FATIGUE NOTCH SENSITIVITY

In ductile materials, the crack will start at some point that causes a stress concentration. The diagram shows the stress concentration at the corner of a groove.

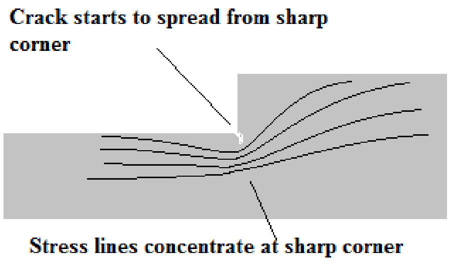


Figure 8

The ratio of the raised stress level to the normal stress level is called the stress concentration factor.

$$k_f = \frac{\sigma}{\sigma_o}$$

There are ways of determining values of k_f for specified cases but this is not covered here.

WORKED EXAMPLE No.2

The fatigue strength of a material in a standard test for a specified number of cycles is 250MPa. The material has a surface notch with a sensitivity factor of 1.4. Calculate the fatigue strength in this case.

SOLUTION

$$\sigma = \sigma_o / k_f = 250 / 1.4 = 178.6 \text{ MPa}$$

WORKED EXAMPLE No.3

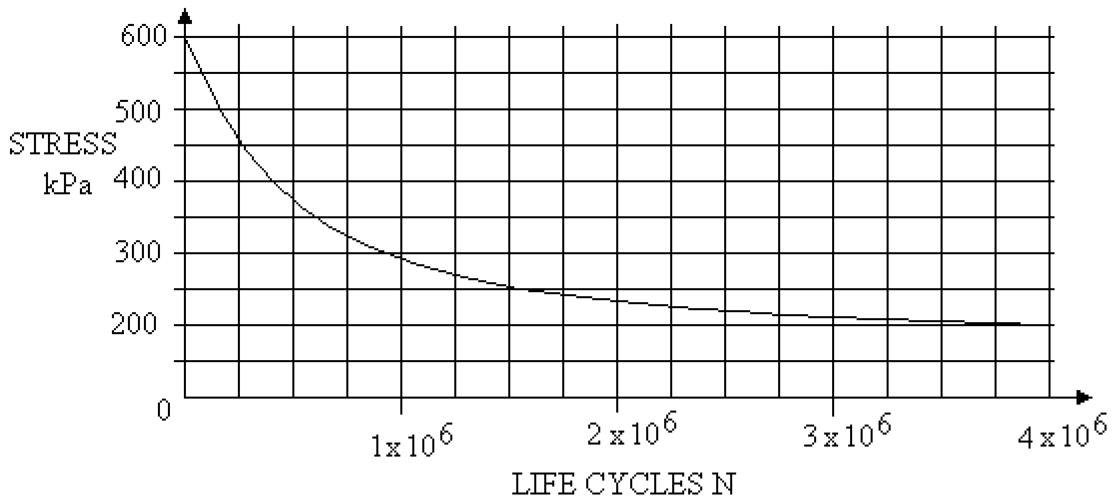
A shaft 50 mm diameter is subject to a bending moment of 3000 Nm. On the surface, there is a notch with a stress concentration factor of 1.6. Calculate the stress produced at this notch.

SOLUTION

$I = \pi D^4/64 = \pi(0.05)^4/64 = 306.8 \times 10^{-9}\ m^4$
$\sigma_o = My/I = 3000 \times 0.025/ 30.68 \times 10^{-6} = 244.5$ MPa
$\sigma = \sigma_o \times k = 244.5 \times 1.6 = 391.1$ MPa

WORKED EXAMPLE No.4

The S – N graph shown is for a certain material. Determine the stress level that will produce a life cycle of 750 000. State the endurance limit of this material.



SOLUTION

From the graph the stress level corresponding to 750 000 cycles is 325 kPa. The endurance limit is 200 kPa.

OTHER FACTORS AFFECTING THE FATIGUE LIFE

Fatigue failure may be accelerated by any of the following:

- Stress concentrations factor
- The way the stress fluctuates
- Corrosion
- Residual surface stress
- Surface finish
- Temperature
- Bulk mass (size) of the component

Stress concentrations were mentioned earlier and are caused by keyways, holes, grooves, undercuts, corners or any surface mark.

Corrosion takes many forms and weakens the metal. Surface deterioration may set up stress raising factors. Corrosion of some metals spreads along the grain boundary and so weakens the material. It has been known for a component to fail in fatigue because a chemical marker had been used to write part numbers on the surface and the chemicals etched into the surface and weakened the grain boundaries in that region.

Residual surface stresses can be set up by bending the material thus leaving a permanent stress in it. If the surface has a residual compressive stress, this is beneficial and may be produced by shot blasting or peening.

If a surface is very smooth, there are no points for a crack to start and no stress raisers. Polishing a component improves its fatigue life. For example, the connecting rods on racing car engines are designed to have the minimum mass possible and so are designed with a very small stress safety margin. This would leave them prone to fatigue failure and polishing them makes fatigue failure less likely. On the other hand, rough surface finishes say from turning on a lathe, reduce the fatigue life. Components have been known to fail in fatigue simply because a part number was engraved on the polished surface.

Hot temperatures cause surface oxidation and degradation and so reduces the fatigue life. Thermal expansion and contraction is itself a cause of fatigue stress For example, the leading edges of aeroplanes get hot in flight and cool at other time causing expansion and contraction. Aeroplane body panels are often shaped by shot blasting so inducing a compressive stress on the surface to counteract fatigue.

The endurance limit of a component in service may be quite different from that found by a standard test. If we assume a constant factor may be used to adjust for each affect then:

$\sigma_{ES} = k_1\ k_2\ k_3\ k_4\ \sigma_E$

k_1 – surface finish factor
k_2 – size factor
k_3 – load factor
k_4 – temperature factor

STRESS PATTERN

Consider a sinusoidal stress variation but with a mean other than zero. The fatigue strength is significantly reduced when the mean is tensile.

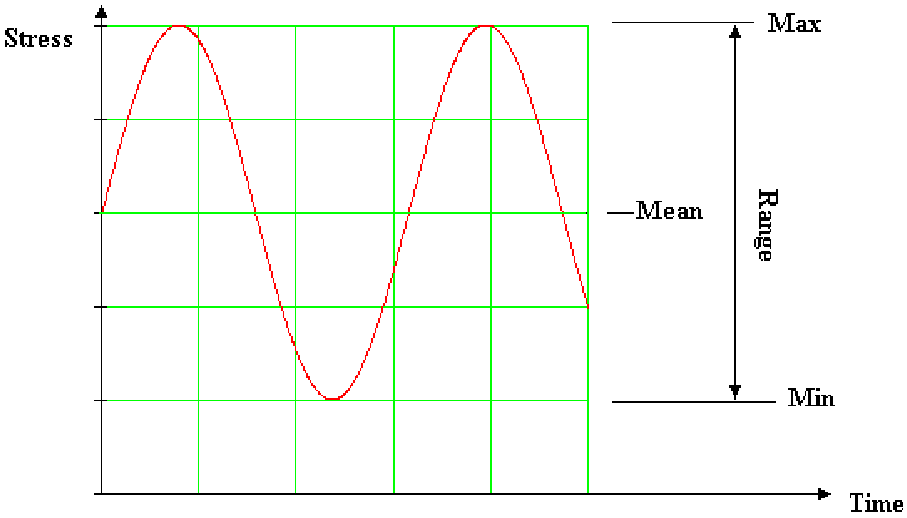


Figure 9

σ_m = mean stress = $(\sigma_{max} + \sigma_{min})/2$
σ_a = stress amplitude$(\sigma_{max} - \sigma_{min})/2$

The way that these quantities affect the endurance limit is the subject of much research. One line of research has suggested the following formula.

$$\frac{1}{SF} = \frac{k_f \sigma_a}{\sigma_{ES}} + \frac{\sigma_m}{\sigma_u}$$

SF is the safety factor and if this is larger than 1 then the component will not fail in fatigue.

WORKED EXAMPLE No.5

A component has been machined from a 38 mm diameter round steel bar. The steel has a normal endurance limit of 345 MPa and a tensile strength of 690 MPa. It is subjected to fluctuating tensile load that varies from 0 to 70 kN. At one point on the component, there is a feature with a stress concentration factor of 1.85. The other factors are:
k_1 – surface finish factor = 0.797
k_2 – size factor = 1
k_3 – load factor = 0.923
k_4 – temperature factor = 1

Determine the safety factor.

SOLUTION

Area = $\pi D^2/4 = \pi \times 0.038^2/4 = 1.134 \times 10^{-3}$ m^2
σ_{max} = Force/Area = 70 000/ 1.134 x 10^{-3} = 61.722 MPa
σ_{min} = 0
σ_a = (61.722 - 0)/2 = 30.86 MPa
σ_m = (61.722 + 0)/2 =30.86 MPa
$\sigma_{ES} = k_1\, k_2\, k_3\, k_4\, \sigma_E$ = 0.797 x 1 x 0.923 x 1 x 345 = 253.8 MPa

$$\frac{1}{SF} = \frac{k_f \sigma_a}{\sigma_{ES}} + \frac{\sigma_m}{\sigma_u} = \frac{1.85 \times 30.86}{253.8} + \frac{30.86}{690} = 0.27$$
$$SF = 3.71$$

SELF-ASSESSMENT EXERCISE

1. Read up as much as you can find from any source about fatigue. Write a short dissertation on fatigue. In your dissertation, you should do the following.

- ***Describe the cause of fatigue in materials.***
- ***Describe the factors that are likely to make fatigue failure more likely.***
- ***Describe examples of engineering structures and/or components in which fatigue is an important design consideration.***

2. A beam is made from a round steel bar 28 mm diameter. At a point on the beam there is a small groove with a stress concentration factor of 1.18. At the same point, there is a bending moment that fluctuates from 565 Nm to 198 Nm producing tensile stress.

The normal endurance limit is 207 MPa and the tensile strength is 800 MPa. There are no other factors to be considered.

Calculate the following.

i. The maximum and minimum bending stress. (262.2 MPa and 91.9 MPa)
ii. The stress amplitude. (85.15 MPa)
iii. The mean stress. (177 MPa)
iv. The fatigue safety factor. (1.42)