Q 82003
The temperature $\theta(\mathrm{s})$ of a room depends on the heat energy input $\Phi(\mathrm{s})$ and the transfer functio relating them is:

$$
\frac{\Phi(\mathrm{s})}{\theta(\mathrm{s})}=\frac{8}{s^{2}+8 \mathrm{~s}+80}
$$

A PID controller is used with unity gain position feedback. The heater transfer function is 5 . The controller gain is set to 2 .
(a) Calculate the value of derivative time control required to give a closed loop damping ratio of unity if there is no integral action.
(b) Calculate the limiting value of integral time that may be used together with the derivative time already found before instability occurs.

First construct a block diagram

$\mathrm{G}_{1}$ with no integral control is $2\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)$
The open loop transfer function is

$$
\mathrm{G}_{\mathrm{ol}}=\frac{\theta_{\mathrm{o}}}{\theta_{e}}(\mathrm{~s})=\mathrm{G}_{1} \mathrm{G}_{2} \mathrm{G}_{3}=2\left(1+\mathrm{s} \tau_{\mathrm{d}}\right) \times 5 \times \frac{8}{\mathrm{~s}^{2}+8 \mathrm{~s}+80}=\frac{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}{\mathrm{s}^{2}+8 \mathrm{~s}+80}
$$

The closed loop transfer function is $G_{c l}=\frac{G_{o l}}{1+G_{o l}}$ or $\frac{1}{1+\frac{1}{G_{o l}}}$
$\mathrm{G}_{\mathrm{cl}}=\frac{\theta_{\mathrm{o}}}{\theta_{\mathrm{i}}}=\frac{1}{1+\frac{1}{\frac{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}{\mathrm{s}^{2}+8 \mathrm{~s}+80}}}=\frac{1}{1+\frac{\mathrm{s}^{2}+8 \mathrm{~s}+80}{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}}=\frac{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)+\mathrm{s}^{2}+8 \mathrm{~s}+80}$
$\mathrm{G}_{\mathrm{cl}}=\frac{\theta_{\mathrm{o}}}{\theta_{\mathrm{i}}}=\frac{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}{\left(80+80 \mathrm{~s} \tau_{\mathrm{d}}\right)+\mathrm{s}^{2}+8 \mathrm{~s}+80}=\frac{80\left(1+\mathrm{s} \tau_{\mathrm{d}}\right)}{\mathrm{s}^{2}+\left(8+80 \tau_{\mathrm{d}}\right) \mathrm{s}+160}$
The characteristic equation is $\quad s^{2}+\left(8+80 \tau_{d}\right) s+160=0$
This could also be found from $1+\mathrm{G}_{\mathrm{ol}}=0$

$$
\begin{aligned}
& 1+\frac{80\left(1+s \tau_{d}\right)}{s^{2}+8 s+80}=0 \\
& s^{2}+8 s+80+80\left(1+s \tau_{d}\right)=0 \\
& s^{2}+s\left(8+80 \tau_{d}\right)+160=0
\end{aligned}
$$

$\omega_{\mathrm{n}}=\sqrt{ } 160=12.65 \mathrm{rads} / \mathrm{s} \quad 2 \delta \omega_{\mathrm{n}}=8+80 \tau_{\mathrm{d}}$
$\delta=1 \quad 2 \omega_{\mathrm{n}}=25.3=8+80 \tau_{\mathrm{d}}$
$17.3=80 \tau_{\mathrm{d}} \quad \tau_{\mathrm{d}}=0.216$
Plotting the root locus confirms this.
The roots of $\mathrm{s}^{2}+\left(8+80 \tau_{\mathrm{d}}\right) \mathrm{s}+160=0$ are $-4-40 \tau_{d} \pm 4 \sqrt{-9+20 \tau_{d}+100 \tau_{d}^{2}}$

Remember that $\delta=\cos (\alpha)$ and so is the point where $\alpha=0^{\circ}$ (measured from the negative real
 axis)and is the breakaway point which occurs at $\tau$ $=0.216$.
(b) $\mathrm{G}_{1}$ with integral control is $2\left\{1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT} \mathrm{d}_{\mathrm{d}}\right\}$

The open loop transfer function is
$\mathrm{G}_{\mathrm{ol}}=\frac{\theta_{\mathrm{o}}}{\theta_{e}}(\mathrm{~s})=\mathrm{G}_{1} \mathrm{G}_{2} \mathrm{G}_{3}=2\left(1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT}_{\mathrm{d}}\right) \times 5 \times \frac{8}{\mathrm{~s}^{2}+8 \mathrm{~s}+80}=\frac{80\left(1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT}_{\mathrm{d}}\right)}{\mathrm{s}^{2}+8 \mathrm{~s}+80}$
The closed loop transfer function is $\mathrm{G}_{\mathrm{cl}}=\frac{1}{1+\frac{1}{\mathrm{G}_{\mathrm{ol}}}}$
$\mathrm{G}_{\mathrm{cl}}=\frac{\theta_{\mathrm{o}}}{\theta_{\mathrm{i}}}=\frac{1}{1+\frac{1}{\frac{80\left(1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT}_{\mathrm{d}}\right)}{\mathrm{s}^{2}+8 \mathrm{~s}+80}}}=\frac{1}{1+\frac{\mathrm{s}^{2}+8 \mathrm{~s}+80}{80\left(1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT}_{\mathrm{d}}\right)}}=\frac{80\left(1+\mathrm{s} \mathrm{\tau}_{\mathrm{d}}\right)}{80\left(1+\frac{1}{\mathrm{sT}_{\mathrm{i}}}+\mathrm{sT}_{\mathrm{d}}\right)+\mathrm{s}^{2}+8 \mathrm{~s}+80}$
$G_{c l}=\frac{\theta_{o}}{\theta_{i}}=\frac{80\left(s+\frac{1}{T_{i}}+s^{2} T_{d}\right)}{\left(80 s+\frac{80}{T_{i}}+80 s^{2} \tau_{d}\right)+s^{2}+8 s+80}=\frac{80\left(s+\frac{1}{s T_{i}}+s^{2} T_{d}\right)}{s^{2}\left(1+80 \tau_{d}\right)+s(88)+80+\frac{80}{T_{i}}}$
The characteristic equation is

