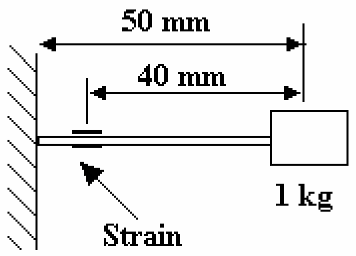
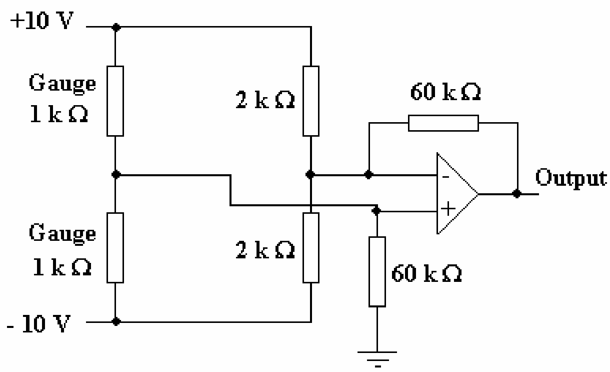
Q12 An accelerometer consists of a 1 kg mass mounted on the end of a cantilever as shown. Two active strain gauges with a gauge factor of 2 are sited as shown. The cantilever is 10 mm wide and 1 mm thick.



(a) Calculate the natural frequency of the system.

(b) The strain gauges are connected to an operational amplifier as shown. Find the zero frequency acceleration responsivity and the amplifier output with beam critically damped.



(c) Find the maximum impact acceleration given that the amplifier limit is ±10V.

The relationship between the applied force F and the deflection of the mass δ is $\delta = 4FL^3/EBD^3$
B is the width, D the thickness and L the length. E is the modulus of elasticity and is 200 GPa.

The stress on the surface of the cantilever is given by $\sigma = 6Fx/bd^2$ where x is the distance from the mass.

SOLUTION

I have to admit that this question is as clear as mud and I have a problem understanding the terminology and in finding the gain of the amplifier.

(a) The natural frequency of any spring mass system is given by $\omega_n^2 = k/M$ where k is the spring stiffness.

$$k = \frac{F}{\delta} = \frac{EBD^3}{4L^3} = \frac{200\text{x}10^9 \text{ x } 0.01 \text{ x } 0.001^3}{4 \text{ x } 0.05^3} = 4000 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{4000}{1}} = 62.246 \text{ rad/s} \quad f_n = \frac{\omega_n}{2\pi} = 10.067 \text{ Hz}$$

The natural frequency is 10.067 Hz

(b) Responsivity is another name for sensitivity and some sources state this is given by :-
s = deflection/acceleration
s = δ/a but from Newton's second law F = M a and the force is F = k δ so $\delta/a = M/k = 1/\omega_n^2$
s = δ/a = M/k = 1/4000 = 250 x 10^{-6} s^2

It is arguable that the examiner is looking for the voltage output per unit acceleration.

The operational amplifier needs analysing next. I can see no way to calculate the gain of the amplifier. The examiner states that the circuit is standard but strain gauge amplifiers usually have a resistor between minus and 0 V which fixes the gain or a 1 kΩ resistor in series with the input terminals. In this case the gain would be 60. This figure will be used.

STRAIN GAUGE BRIDGE

$$\text{stress} = \sigma = \frac{6Fx}{bd^2} = \frac{6 \ \text{x} \ 0.04F}{0.01 \ \text{x} \ 0.001^2} = 24 \ \text{x} 10^6 F$$

The strain is ε = stress/Modulus = σ/E =24 x 10^6 F /200 x 10^9 = 120 x10^{-6} F
The change in resistance of a strain gauge is Gε
G is the gauge factor = 2 so the electrical strain is ΔR/R = 240 x 10^{-6} F
R = 1 kΩ so ΔR = 1000 x 240 x 10^{-6} F = 240 x 10^{-3} F

With two active gauges as shown we get twice the output of a single gauge.
Let one increase and the other decrease by ΔR. The voltage at the junction relative to minus terminal is :- V_S(R - ΔR)/2R = 20 (1000 - 240 x 10^{-3} F)/2000 = 10 ± 0.0024F Volts

The voltage difference at the input terminals is ± 0.0024F
The voltage output is 6 times larger so V_o = 0.144 F

Since F = Mass x Acceleration then V_o = 0.144 M a = 0.14 a

The overall sensitivity is V_o /a = 0.14 volts per unit acceleration. This may be the answer required.

With zero frequency and critical damping, the only force on the beam is the weight of 1 kg or 9.81 N. This will produce an output unless there is a system to zero the accelerometer with no acceleration present. If this is what the examiner is looking for, then the question is worded in a manner to confuse us all.

V_o = 0.144 F = 0.144 x 9.81 = ±1.412 Volts the polarity depending which way around the gauges are connected.

(c) The maximum output of the amplifier is ±10 V
a = 10/0.14 = 71.42 m/s^2
But if there is an offset voltage of 1.412 V the maximum acceleration will be more in one direction and less in the other.