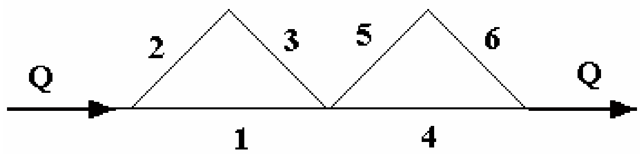
APPLIED FLUID MECHANICS D203 Q8 2004

(a) The figure shows a network of pipes transporting water at a flow rate Q from the inlet node to the outlet node. The individual pipes in the network are numbered as shown and the friction head loss in the pipes is given by $(h_f)_i = k_i Q_i^2$ for I = 1,2 …..6

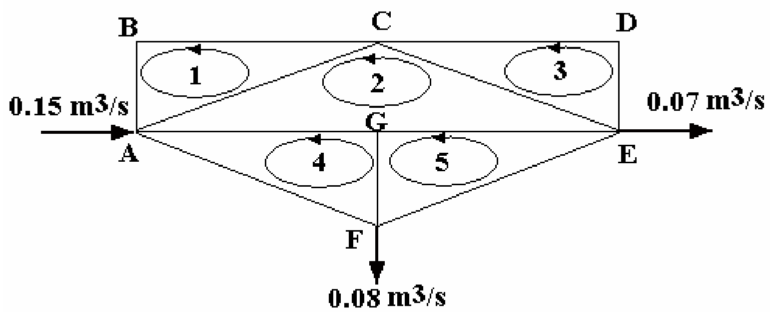
Derive an expression for the head loss k factor of an equivalent single pipe connected between the same inlet and outlet nodes and transporting the same total flow Q



(b) A multi-loop network is shown in the second figure. The friction head loss factor k for each pipe and the elevation of each node above a common datum are Given in the table.

(i) Use the result from part (a) to simplify the pipe network.
(ii) Use two iterations of the Hardy Cross method of solution to estimate the flow distribution in the simplified pipe network.
(iii)If the pressure head at outlet E must be at least 20 m, calculate the minimum pressure head required at inlet node A.

Note. The Hardy Cross flow correction equation is $\Delta Q = \dfrac{-\sum h_f}{2\sum \frac{h_f}{Q}}$



The loops have been added to aid solution

Pipe	AB	AC	BC	CD	CE	DE	AG	AF	EF	GF	GE
k s^2/m^5	100	120	150	100	140	160	150	180	150	150	180

Node	A	B	C	D	E	F	G
Elevation m	12	14	10	8	10	8	10

(a) For the first half of the network (1,2 and 3)

$\Delta h_a = Q_2^2(k_2 + k_3) \qquad Q_2^2 = \dfrac{\Delta h_a}{k_2 + k_3}$

$\Delta h_a = Q_1^2(k_1) \qquad Q_1^2 = \dfrac{\Delta h_a}{k_1} \qquad Q = Q_1 + Q_2 = \sqrt{\dfrac{\Delta h_a}{k_2 + k_3}} + \sqrt{\dfrac{\Delta h_a}{k_1}}$

$$Q = Q_1 + Q_2 = \sqrt{\Delta h_a}\left\{\sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}}\right\} = \Delta h_a \left\{\sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}}\right\}^2 = \frac{\Delta h_a}{k_a}$$

$$k_a = \frac{1}{\left\{\sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}}\right\}^2}$$

For the second half (4, 5 and 6) an identical result is obtained

$$Q^2 = \Delta h_b \left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2 = \frac{\Delta h_b}{k_b} \text{ and } k_b = \frac{1}{\left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2}$$

The pressure head drop over the whole network is hence

$$\Delta h = \Delta h_a + \Delta h_b = kQ^2 \text{ where } k = k_a + k_b$$

(b) The top half of the network is the same as part (a) so we may evaluate k for this part.

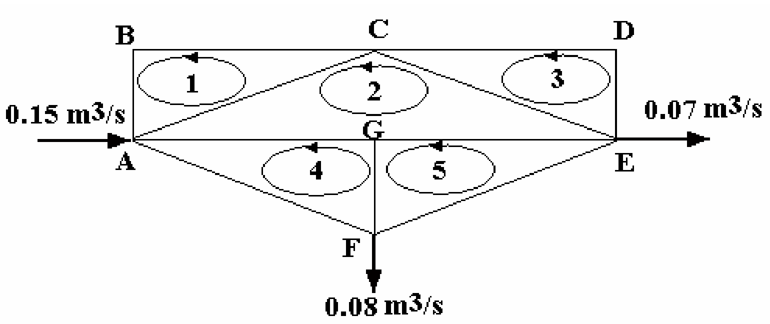
$$k = \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{k_5 + k_6}} + \sqrt{\frac{1}{k_4}} \right\}^2}} + \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{k_2 + k_3}} + \sqrt{\frac{1}{k_1}} \right\}^2}}$$

$$k = \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{100 + 160}} + \sqrt{\frac{1}{140}} \right\}^2}} + \frac{1}{\sqrt{\left\{ \sqrt{\frac{1}{100 + 150}} + \sqrt{\frac{1}{120}} \right\}^2}}$$

$$k = 88.45$$

If the flow law is applied between A and E for the top loop and assuming the friction head is equal to the change in altitude we get:

$Q_a = \sqrt{(\Delta h/k)}$ Δh = difference between A and E $\Delta h = 2$ m
$Q_a = \sqrt{(2/88.45)} = 0.150$ m^3/s which is the same as the total flow at A



Loop 4

Iteration 1 Note h_f is minus if Q is minus

Pipe	Flow(guess)	k	$h_f = kQ^2$	$h_f/Q = kQ$
AF	0.1	180	1.8	18
FG	-0.05	150	-0.375	7.5
GA	0.1	150	1.5	15
Totals			2.925	40.5

$\Delta Q = -(2.925)/(2 \times 40.5) = -0.036$ so Q (FG)=-0.05 - 0.036 =-0.086

LOOP 5

Iteration 1 Note h_f is minus if Q is minus

Pipe	Flow(guess)	k	$h_f = kQ^2$	$h_f/Q = kQ$
GF	-0.086	150	-1.112	12.93
FE	0.1-0.08	150		
EG		180		
Totals				

$\Delta Q = -(-0.075)/(2 \times 40.5) = 0.000926$

Guess Q AF = 0.1 Q FG = 0.05 Q GA = 0.05

h_f (AF) = $180\ (0.1)^2$ =