7 (a) Show that to avoid a shock loss at entry to the runner of an inward radial flow reaction turbine when the blade thickness at inlet to the runner is negligible, the inlet guide vane angle β_1 should have the value given by the expression

$$\beta_1 = \cot^{-1}\left[\frac{2\pi r_1^2 h_1 \omega}{Q} + \cot\alpha_1\right]$$

where r_1 is the runner blade inlet radius, h_1 is the blade width at inlet, α_1 is the runner blade inlet angle (relative to a tangent to the runner), ω is the angular velocity of the runner and Q is the water flow rate through the turbine.

(b) An inward radial flow reaction turbine is supplied with 0.700 m^3/s of water under an effective head of 16 m. The runner is rotating at 300 rev/min and its inner and outer diameters are 0.5 m and 0.75 m respectively. The runner blade width at inlet is 0.1 m and the blade inlet angle is 105° to a tangent to the runner. The flow is discharged radially from the runner to atmospheric pressure. Given that the thickness of the blades at inlet to the runner is negligible and the flow component of velocity is constant through the runner, calculate

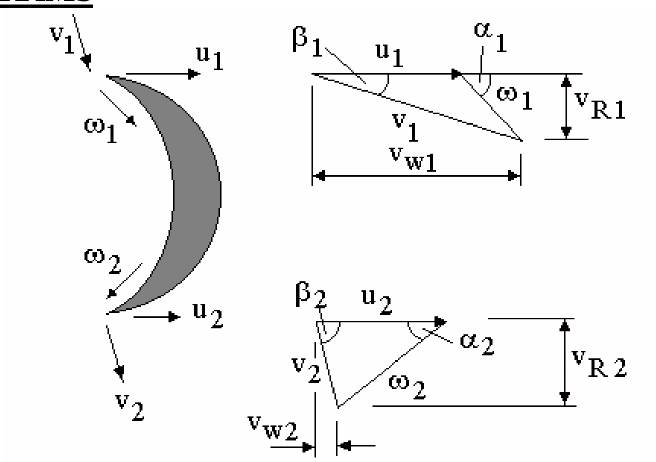
(i) the guide vane inlet angle for no shock loss in the runner
(ii) the runner blade outlet angle
(iii) the output shaft power available from the turbine if the mechanical efficiency is 93%
(iv) the overall efficiency of the turbine.

Note: $$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

SOLUTION

Comment – this seems a very reasonable question but I don't know why the trig identity is needed.

BLADE VECTOR DIAGRAMS



v_R = radial velocity and is constant

A = circumferential area = $2\pi r h$ $v_R = Q/(2\pi r h)$ h = height of the vane.

$$v_{w1} = u_1 + v_{r1}\cot\alpha_1$$

$$\cot\beta_1 = \frac{v_{w1}}{v_r} = \frac{u_1 + v_{r1}\cot\alpha_1}{v_{r1}} = \frac{u_1}{v_r 1} + \cot\alpha_1 \quad u = \omega r_1 \quad v_{r1} = \frac{Q}{2\pi r_1 h_1}$$

(b) (i) Q = 0.700 m^3/s N = 300 rev/min D_1 = 0.75 m D_2 = 0.5 m Radial discharge

$$\cot\beta_1 = \frac{2\pi r_1^2 h_1 \omega}{Q} + \cot\alpha_1 \qquad \beta_1 = \cot^{-1}\left(\frac{2\pi r_1^2 h_1 \omega}{Q} + \cot\alpha_1\right)$$

$R_1 = 0.375$ $h_1 = 0.1$ m $\alpha_1 = 105°$ $\omega = 2\pi N = 2\pi(300/60) = 31.416$ rad/s

$$\beta_1 = \cot^{-1}\left(\frac{2\pi r_1^2 h_1 \omega}{Q} + \cot\alpha_1\right) = \cot^{-1}\left(\frac{2\pi \times 0.25^2 \times 0.1 \times 31.416}{0.7} + \cot 105^o\right) = \cot^{-1}(3.965 - 0.268)$$

$\beta_1 = \cot^{-1}(3.697)$ $\beta_1 = 15.1^o$

(ii) From the outlet triangle with radial discharge we have
$\tan\alpha_2 = v_{R2}/u_2$ $v_{R2} = v_R = Q/(2\pi r\, h) = 2.971$ m/s
$\omega = 2\pi N = 2\pi \times 300/60 = 10\pi$ rad/s
$u_2 = \omega D_2/2 = 7.854$ m/s

The runner blade outlet angle = $\alpha_2 = 20.7^o$

(iii) DIAGRAM POWER = D.P. = $m\,\Delta(uv_w) = m\,(u_1v_{w1} - u_2v_{w2})$ radial discharge so $u_2v_{w2} = 0$
$u_1 = \omega D_1/2 = 11.78$ m/s $v_{w1} = 10.985$ m/s
D.P. = $m\,(u_1v_{w1}) = \rho Q\, u_1v_{w1} = 997 \times 0.7 \times 11.78 \times 10.985 = 90.32 \times 10^3$ W

The output shaft power available from the turbine if the mechanical efficiency is 93% is

Shaft Power = 0.93 x 90.32 = 84 kW

(iv) Water Power = $mg\Delta H = \rho Qg\,\Delta H = 997 \times 0.7 \times 9.81 \times 16 = 109.5$ kW

The overall efficiency of the turbine = SP/WP = 76.7%