APPLIED FLUID MECHANICS D203 QUESTION 5 2004

(a) Explain how momentum is transferred in the turbulent flow of a fluid and compare this with momentum transfer in laminar flow.

(b) (i) The mean shear stress τ in a turbulent flow in a pipe is given by $\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

where ρ is fluid density, l is the Prandtl mixing length and u is the time-mean velocity at distance y from the pipe wall. Assuming that the mean shear stress τ is equal to the wall shear stress τ_o and taking the mixing length l = 0.4y, show that

$$\frac{u_m - u}{u^*} = 5.75\log_{10}\frac{R}{y}$$

u_m is the maximum velocity in the pipe. $u^* = \sqrt{\frac{\tau_o}{\rho}}$ is the friction velocity and R is the pipe radius.

(ii) Explain why this relationship does not apply close to the pipe wall.

(c) Use calculations to determine the smallest diameter of commercial galvanized steel pipe required to transport water over a horizontal distance of 200m at a flow rate of 0.10 m^3/s if the head loss is not toexceed 10m. Available commercial pipes have diameters of 80, 100, 150, 200, 250 and 300 mm. The roughness factor *k* for galvanized pipe is 0.15 mm.

The Darcy friction factor *f* for the flow may be obtained from the empirical equation

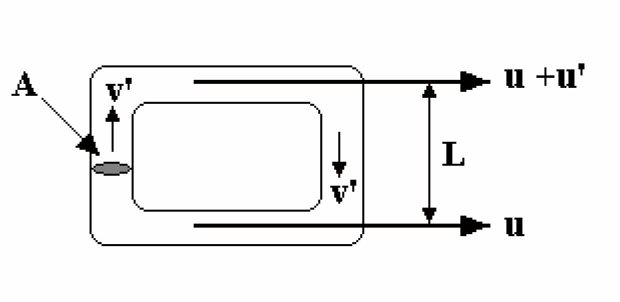
$$\frac{1}{\sqrt{f}} = -0.782\ln\left[\frac{6.9}{Re} + \left(\frac{k/D}{3.71}\right)^{1.11}\right]$$

where Re is the Reynolds Number for the pipe based on diameter.

The Darcy friction factor *f* is used in the following formula for friction head loss in a pipe

$h_f = f\frac{Lv^2}{2gD}$ The symbols have their usual meanings.

(a) In laminar flow friction occurs between parallel layers and momentum is transferred by the drag exerted by one layer on the other. In turbulent flow eddy currents are set up to transfer momentum between layers. An idealized eddy is shown in the diagram as a closed loop of cross sectional area A with a velocity v'. Consider two layers distance L apart. The upper layer moves at velocity (u + u') and the lower layer at u.



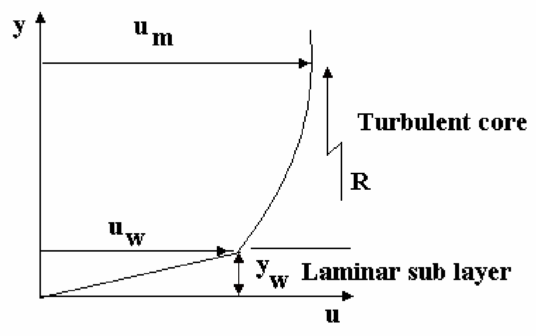
Assuming that u'/L = du/dy u' = L du/dy. The difference in velocity of the layers must be 2v' if no slippage is occurring so u' = 2v' and hence v' = ½ L du/dy.

The mass flow rate within the eddy is ρAv' and an equal mass is transferred from one layer to the other so the total interchange is 2ρAv'

The rate of change of momentum is $2\rho A v' u' = \rho A u'^2$
And this is a force $F = \rho a u'^2$

The shear stress acting on the area A is τ so $2\tau A = \rho A u'^2$ hence $\tau = \frac{1}{2}\rho u'^2 = \tau = \rho l^2 \left(\frac{du}{dy}\right)^2$

b) The diagram shows velocity u plotted against distance y from the wall. Close to the wall is a thin laminar layer so the following only applies to the turbulent core.



$$\tau = \rho l^2\left(\frac{du}{dy}\right)^2 = \rho(ky)^2\left(\frac{du}{dy}\right)^2 = \rho k^2 y^2\left(\frac{du}{dy}\right)^2$$

$$\tau_o = \rho k^2 y^2\left(\frac{du}{dy}\right)^2$$

$$u^* = \sqrt{\frac{\tau_o}{\rho}} = \frac{\sqrt{\rho}\,ky}{\sqrt{\rho}}\frac{du}{dy} = ky\frac{du}{dy} \qquad du = u^*\frac{dy}{ky}$$

Integrate $u = u^*\frac{1}{k}\ln y + C$

At the centre of the pipe where y = R the velocity is u_m

$u_m = u^*\frac{1}{k}\ln R + C$ $\quad C = u_m - u^*\frac{1}{k}\ln R$ and substitute back for C

$$u = u^*\frac{1}{k}\ln y + u_m - u^*\frac{1}{k}\ln R = u_m + u^*\frac{1}{k}\ln\frac{y}{R}$$

$$\frac{u_m - u}{u^*} = u^*\frac{1}{k}\ln\frac{R}{y}$$

put k = 0.4 and note that to convert to $\log_{10}$ we multiply by ln 10

$$\frac{u_m - u}{u^*} = u^*\frac{\ln 10}{0.4}\log_{10}\frac{R}{y} = 5.75u^*\log_{10}\frac{R}{y}$$

(c) $h_f = 10 = f\frac{Lv^2}{2gD} = f\frac{200\,v^2}{2gD}$ $\quad v = Q/A = 4Q/\pi D^2 = 4 \times 0.1/\pi D^2 = 0.127D^{-2}$

$10 = f\frac{200\,(0.127D^{-2})^2}{2gD} = f \times 0.165\,D^{-5}$ $\qquad f = 60.61\,D^5$

$Re = \rho vD/\mu = 997 \times 0.127\ D^{-1}/0.89 \times 10^{-3} = 142269\ D^{-1}$

$$\frac{1}{\sqrt{f}} = -0.782\ln\left[\frac{6.9}{Re} + \left(\frac{k/D}{3.71}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{60.61D^5}} = -0.782\ln\left[\frac{6.9D}{142269} + \left(\frac{0.15 \times 10^{-3}}{3.71D}\right)^{1.11}\right]$$

$$\frac{1}{7.785D^{5/2} \times 0.782} = -\ln\left[648.5 \times 10^{-6}D + \left(40.43 \times 10^{-6}D^{-1}\right)^{1.11}\right]$$

$$0.1642D^{-5/2} = -\ln\left[648.5 \times 10^{-6}D + \left(40.43 \times 10^{-6}D^{-1}\right)^{1.11}\right]$$

Plotting both functions against D we see that a diameter that satisfies both sides is just over 200 mm.