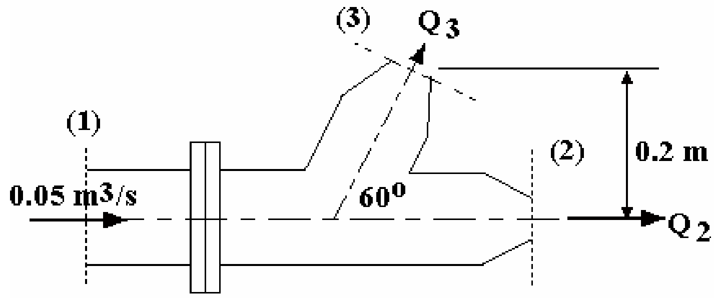
APPLIED FLUID MECHANICS D203 SOLUTIONS 2004 – QUESTION 3

A 150 mm diameter pipe is fitted with a double nozzle with the geometry shown. At section (2) the exit diameter of the nozzle is 80 mm and at section (3) the exit diameter is 100 mm. The pipe and both nozzles lie in the same vertical plane. A steady discharge of 0.05 m^3/s of water from the pipe at section (1) emerges as jets from the nozzles into the surrounding atmosphere. Energy losses in the flow may be assumed to be negligible.

(a) Calculate
  (i) the flow rate at each nozzle exit
  (ii) the water pressure in the pipe at section (1).

(b) The double nozzle fitting is attached to the pipe by a bolted flange. The fitting has a material mass of 5 kg and the water volume within it is 0.005 m^3. Calculate the magnitude and direction of the resultant force applied to the group of flange bolts for the flow conditions described above.



$D_1 = 0.15$m $D_2 = 0.08$m $D_3 = 0.1$m $Q1 = 0.05\ m^3/s$
$p_1 = 400 \times 10^3\ N/m^2$ $p_2 = 395 \times 10^3\ N/m^2$

$$A_1 = \frac{\pi D_1^2}{4} = 0.018\,m^2 \quad A_2 = \frac{\pi D_2^2}{4} = 0.005027\,m^2 \quad A_3 = \frac{\pi D_3^2}{4} = 0.007854\,m^2$$

$$u_1 = \frac{Q_1}{A_1} = 2.829\ m/s$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (2)

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$p_1 + 998\frac{2.829^2}{2} = 0 + 998\frac{u_2^2}{2} \quad p_1 = 499u_2^2 - 3.995 \times 10^3$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (3)

$$p_1 + \rho\frac{u_1^2}{2} = p_3 + \rho\frac{u_3^2}{2} + \rho g z$$

$$p_1 + 998\frac{2.829^2}{2} = p_3 + 998\frac{u_3^2}{2} + 0.2 \times 9.81 \times 998 \quad p_1 = 499u_3^2 - 2.037 \times 10^3$$

$$p_1 = +499u_2^2 - 3.995 \times 10^3 = 499u_3^2 - 2.037 \times 10^3$$

$$u_2^2 = u_3^2 + 3.918$$

CONSERVATION OF MASS

$$\rho Q_1 = 0.05\rho = \rho Q_2 + \rho Q_3 \qquad Q_2 = 0.05 - Q_3 \qquad u_2 A_2 = 0.05 - u_3 A_3$$

$$5.027 \times 10^{-3} u_2 = 0.05 - 7.854 \times 10^{-3} u_3$$

$$u_2 = 9.947 - 1.562u_3 \qquad u_2^2 = 98.94 + 2.441u_3^2 - 31.08u_3$$

$$u_2^2 = 98.94 + 2.441u_3^2 - 31.08u_3 = u_3^2 + 3.918$$

$$1.441u_3^2 - 31.08u_3 + 95.02$$

Solving the quadratic $u_3 = 3.69$ m/s the other solution 17.88 would give a flow larger than Q_1.
$Q_3 = A_3 u_3 = 0.029\ m_3/s$ $Q_2 = Q_1 - Q_3 = 0.021\ m_3/s$

Check $u_2 = \sqrt{u_3^2 + 3.918} = 4.185$ m/s

$Q_2 = A_2 u_2 = 0.021$ m_3/s

(ii) $p_1 = 499u_2^2 - 3.995 \times 10^3 = 4.747 \times 10^3$ N/m^2

FORCES

Momentum at (1) = $\rho A_1 u_1^2 = 998 \times 0.01767 \times 2.829^2 = 141.2$

Momentum at (2) = $\rho A_2 u_2^2 = 998 \times 0.00503 \times 4.185^2 = 87.9$

Momentum at (3) = $\rho A_3 u_3^2 = 998 \times 0.007854 \times 3.69^2 = 106.6$

Resolve vertically and horizontally

Horizontal Momentum = 106.6 cos 60^o =53.3

Vertical Momentum = 106.6 sin 60^o = 92.3

PRESSURE FORCES

Pressure force at (1) = $p_1 A_1 = 83.9$ N

Pressure force at (2) and (3) are zero since gauge pressures are being used.

Weight = 5 x 9.81 + weight of water = 5 x 9.81 + 0.005 x 998 = 54 N↓

TOTALS on FLANGE

HORIZONTAL $\Delta mv + \Delta pA = -141.2 + 87.9 + 53.3 - 83.9 = 83.9$ N (to right)

VERTICAL $\Delta mv - W = -92.3 - 54 = -146.3$ N (Down no pressure force)

Total = $\sqrt{(83.9^2 + 146.3^2)} = 168.6$ N

Angle = $\tan^{-1}(146.3/83.9) = 60^o$ to vertical