APPLIED FLUID MECHANICS D203 SOLUTIONS 2004

1. The energy per unit mass (gH) given to a liquid by a centrifugal pump is known to be dependant on the diameter D, impeller speed N, liquid volumetric flow rate Q, liquid density ρ, liquid dynamic viscosity μ and the power supplied to the pump P.

(a) Use dimensional analysis applied to geometrically similar pumps to derive the functional expression

$$\left(\frac{gH}{N^2D^2}\right) = \varphi\left(\frac{Q}{ND^3}\right)\left(\frac{P}{\rho N^3D^5}\right)\left(\frac{\mu}{N\rho D^2}\right)$$

SOLUTION

$(gH) = f(D, N, Q, \rho, \mu, P)$ It is normal to consider gΔH as one quantity.

There are 7 quantities and 3 dimensions so there are four dimensionless groups Π_1, Π_2 Π_3 and Π_4. First form a group with gH and ρND

$$gH = \varphi(\rho ND) = \Pi_1\rho^aN^bD^c$$

$$L^2T^{-2} = (ML^{-3})^a(T^{-1})^b(D^1)^c$$

Mass $0 = a$ Time $-2 = -b$ $b = 2$ Length $2 = -3a + c = c$ $c = 2$

$$gH = \Pi_1N^2D^2 \qquad \Pi_1 = \frac{gH}{N^2D^2}$$

Next repeat the process between Q and ρND

$$Q = \varphi(\rho ND) = \Pi_2\rho^aN^bD^c$$

$$M^3T^{-1} = (ML^{-3})^a(T^{-1})^b(D^1)^c$$

Time $-1 = -b$ $b = 1$ Mass $0 = a$ Length $3 = -3a + c$ $c = 3$

$$Q = \Pi_2\rho^0N^1D^3 \qquad \Pi_2 = \frac{Q}{ND^3}$$

Next repeat the process between P and ρND

$$P = \varphi(\rho ND) = \Pi_3\rho^aN^bD^c$$

$$M^1L^2T^{-3} = (ML^{-3})^a(T^{-1})^b(D^1)^c$$

Mass $1 = a$ Time $-3 = -b$ $b = 3$ Length $2 = -3a + c = -3 + c$ $c = 5$

$$P = \Pi_3\,\rho^1N^3D^5 \qquad \Pi_3 = \frac{P}{\rho N^3D^5}$$

Next repeat the process between μ and ρND

$$\mu = \varphi(\rho ND) = \Pi_4\rho^aN^bD^c$$

$$M^1L^{-1}T^{-1} = (ML^{-3})^a(T^{-1})^b(D^1)^c$$

Mass $1 = a$ Time $-1 = -b$ $b = 1$ Length $-1 = -3a + c$ $c = 2$

$$\mu = \Pi_4\rho^1N^1D^2 \qquad \Pi_4 = \frac{\mu}{\rho ND^2}$$

Finally the complete equation is $\left(\frac{gH}{N^2D^2}\right) == \varphi\left(\frac{Q}{ND^3}\right)\left(\frac{P}{\rho N^3D^5}\right)\left(\frac{\mu}{N\rho D^2}\right)$

(b) Using these groups, derive the dimensionless specific speed of a pump and describe the principal applications of specific speed.

The specific speed is a parameter used for pumps and turbines to determine the best design to match a given pumped system. The formula may be derived from consideration of the pump geometry or by dimensional analysis. It is found by equating the diameter between Π_1 and Π_2 (the head and flow coefficients).

$$\Pi_2 = \frac{Q}{ND^3} \quad D = \left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} \quad \Pi_1 = \frac{gH}{N^2D^2} \quad D = \left(\frac{gH}{N^2\Pi_1}\right)^{\frac{1}{2}}$$

Equating
$$\left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} = \left(\frac{gH}{N^2\Pi_1}\right)^{\frac{1}{2}} \qquad \frac{1}{N}\left(\frac{gH}{\Pi_1}\right)^{\frac{1}{2}} = \frac{Q^{\frac{1}{3}}}{\Pi_2^{\frac{1}{3}}N^{\frac{1}{3}}}$$

$$\frac{(H)^{\frac{1}{2}}}{Q^{\frac{1}{3}}N^{\frac{2}{3}}} = \frac{\Pi_1^{\frac{1}{2}}}{\Pi_2^{\frac{1}{3}}g^{\frac{1}{2}}} = \text{constant} \qquad \frac{(H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}} = N^{\frac{2}{3}}$$

$$\left[\frac{(H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}}\right]^{\frac{3}{2}} = N = \frac{(H)^{\frac{3}{4}}}{K^{\frac{1}{2}}Q^{\frac{1}{2}}} \qquad \frac{NQ^{\frac{1}{2}}}{(H)^{\frac{3}{4}}} = K^{-\frac{1}{2}} = \text{constant}$$

$$N_s = \frac{NQ^{\frac{1}{2}}}{(H)^{\frac{3}{4}}}$$

Traditionally the units used are rev/min for speed, m^3/s for flow rate and metres for head. It may be regarded as the speed to produce a unit flow at unit head.

(c) A centrifugal pump operating at its best efficiency point provides 0.28 m^3/s of water at a head of 2 m and consumes 6.3 kW of power. A geometrically similar pump with an impeller 40% larger is to operate at 20% higher speed. Calculate the flow, head and power required for this second pump running at its best efficiency.

We must have dynamic similarity using $D_1 = 1$ and $D_2 = 1.4$ $N_1 = 1$ $N_2 = 1.2$

First using the flow coefficient

$$\frac{Q_1}{N_1D_1^3} = \frac{Q_2}{N_2D_2^3} \qquad Q_2 = \frac{Q_1N_2D_2^3}{N_1D_1^3} = \frac{0.28 \times 1.2 \times 1.4^3}{1 \times 1^3} = 0.922 \text{ m}^3/\text{s}$$

Next using the head coefficient

$$\frac{gH_1}{N_1^2D_1^2} = \frac{gH_2}{N_2^2D_2^2} \qquad H_2 = \frac{H_1N_2^2D_2^2}{N_1^2D_1^2} = \frac{2 \times 1.2^2 \times 1.4^4}{1} = 5.645 \text{ m}$$

Next using the power coefficient

$$\frac{P_1}{\rho N_1^3D_1^5} = \frac{P_2}{\rho N_2^3D_2^5} \qquad P_2 = \frac{P_1N_2^3D_2^5}{N_1^3D_1^5} = \frac{6.3 \times 1.2^3 \times 1.4^5}{1} = 58.55 \text{ kW}$$