D203 FLUID MECHANICS

Q5 1999

For an element of fluid the force balance is:

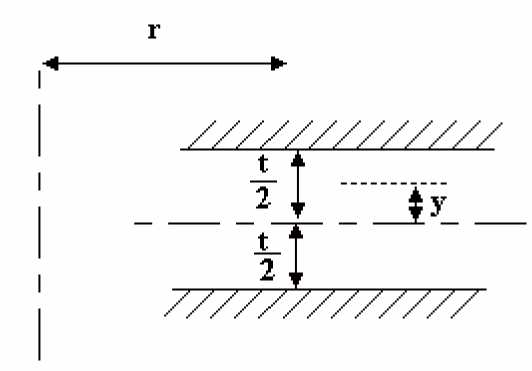
$$d\tau\, dr = dp\, dy \qquad \frac{dp}{dr} = \frac{d\tau}{dy} = \frac{d\left(\mu \frac{du}{dy}\right)}{dy} = \mu \frac{d^2u}{dy^2}$$

INTEGRATE

$$y\frac{dp}{dr} = \mu\frac{du}{dy} + A$$

INTEGRATE

$$\frac{y^2}{2}\frac{dp}{dr} = \mu u + Ay + B$$



Boundary conditions are at y = ± b/2 u = 0

Put y = t/2

$$\frac{(t/2)^2}{2}\frac{dp}{dr} = 0 + At/2 + B..........(1)$$

Put y = -t/2

$$\frac{(-t/2)^2}{2}\frac{dp}{dr} = 0 - At/2 + B..........(2)$$

Add (1) + (2)

$$(t/2)^2\frac{dp}{dr} = 2B \qquad B = \frac{t^2}{8}\frac{dp}{dr}$$

Substitute into (1) $\frac{t^2}{8}\frac{dp}{dr} = 0 + \frac{At}{2} + \frac{t^2}{8}\frac{dp}{dr}$

It follows that A = 0

$$\frac{y^2}{2}\frac{dp}{dr} = \mu u + \frac{t^2}{8}\frac{dp}{dr}$$

$$\frac{dp}{dr}\left\{\frac{y^2}{2} - \frac{t^2}{8}\right\} = \mu u$$

$$u = \frac{dp}{dr}\frac{1}{8\mu}\left\{4y^2 - t^2\right\}$$

For an elementary ring radius r and height dy

dA = 2πr dy dQ = u 2πr dy

$$dQ = \frac{dp}{dr}\frac{1}{8\mu}\left\{4y^2 - t^2\right\} x\ 2\pi r\, dy$$

$$dQ = \frac{dp}{dr}\frac{2\pi r}{8\mu}\left\{4y^2 dy - t^2 dy\right\}$$

Integrate with respect to y

$$Q = \frac{dp}{dr}\frac{2\pi r}{8\mu}\left[\frac{4y^3}{3} - t^2 y\right]_{-\frac{t}{2}}^{\frac{t}{2}}$$

$$Q = \frac{dp}{dr}\frac{2\pi r}{8\mu}\left[\left(-\frac{4t^3}{24} + \frac{t^3}{2}\right) - \left(\frac{4t^3}{24} - \frac{t^3}{2}\right)\right]$$

$$Q = \frac{dp}{dr}\frac{\pi r}{4\mu} x \frac{2t^3}{3}$$

$$\frac{dr}{r} = dp\frac{\pi t^3}{6\mu\mu}$$

$$\int_{R_i}^{R_o}\frac{dr}{r} = \frac{\pi t^3}{6\mu\mu}\int_p^0 dp$$

Integrate

$$\ln\left(\frac{R_o}{R_i}\right) = \frac{\pi t^3}{6\mu\mu}\,p$$

$$Q = \frac{\pi t^3}{6\mu}\frac{p}{\ln\left(\frac{R_o}{R_i}\right)}$$

$t = 5$ mm $R_o = 0.15$ m $R_i = 0.025$ m $\rho = 800$ kg/m^3 $\mu = 0.25$ Ns/m^2 $u_m = 5$ m/s

$Q = A\,u_m = \pi \text{ x } 0.025^2 \text{ x } 5 = 9.817 \text{ x } 10^{-3}$ m^3/s

$$p = \frac{Q\ln\left(\frac{R_o}{R_i}\right) x\, 6\mu}{\pi t^3} = 9.817 \text{ x } 10^{-3}\,\frac{\ln\left(\frac{0.15}{0.025}\right) x\, 6 \text{ x } 0.25}{\pi \text{ x } 0.005^3} = 67.19 \text{ kPa}$$

Max velocity at y = 0

$$\frac{dp}{dr} = -\frac{67190}{0.15 - 0.025} = 537 \text{ x } 10^{-3}$$

$$u = \frac{1}{8\mu}\frac{dp}{dr}\left(4y^2 - t^2\right) = \frac{1}{8 \text{ x } 0.25}(-537 \text{ x } 10^{-3})\left(-0.005^2\right) = 6.72 \text{ m/s}$$