D203 FLUID MECHANICS

Q3 1999

$$u = u_x \frac{y}{\delta} \quad \frac{u}{u_x} = \frac{y}{\delta}$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{u_x}\right) dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy$$

$$\delta^* = \left[y - \frac{y^2}{2\delta} \right]_0^{\delta}$$

$$\delta^* = \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\theta = \int_0^{\delta} \left(\frac{u}{u_x} - \left(\frac{u}{u_x} \right)^2 \right) dy = \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^2}{\delta^2} \right) dy$$

$$\theta = \left[\frac{y^2}{2\delta} - \frac{y3}{3\delta^2} \right]_0^{\delta} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$m = \rho A u_o = \rho\ 2h\ u_o$

At a point in the pipe $m = \rho A u_x$
$A = 2h - 2\delta^*$

$\rho\ 2h\ u_o = \rho(2h - 2\delta^*)u_x$
$2h\ u_o = (2h - 2\delta/2)u_x$
$2h\ u_o = (2h - \delta)u_x$
$2h\ u_o = 2h\ u_x - \delta\ u_x$
$2\ u_o = 2\ u_x - \delta\ u_x/h$
$(\delta/h)\ u_x = 2\ u_x - \delta\ u_o$

$$\frac{\delta}{h} u_x = 2u_x - 2u_0 = 2((u_x - u_0)$$

$$p_o + \frac{\rho u_o^2}{2} = p_f + \frac{\rho u_x^2}{2}$$

$$p_o - p_f = \frac{\rho u_x^2}{2} - \frac{\rho u_o^2}{2} = \frac{\rho}{2}\left(u_x^2 - u_o^2\right)$$

At point $\delta = h \quad 2\left(1 - \frac{u_o}{u_x}\right) = 1 \quad u_x = 2\ u_o$

$$p_o - p_f = \frac{\rho}{2}\left(4u_o^2 - u_o^2\right)$$

$$\frac{p_o - p_f}{\rho u_o^2/2} = 3$$