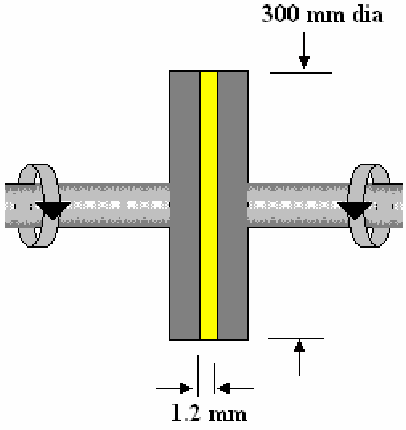
A simple fluid coupling consists of two parallel round discs of radius R separated by a a gap h. One disc is connected to the input shaft and rotates at ω_1 rad/s. The other disc is connected to the output shaft and rotates at ω_2 rad/s. The discs are separated by oil of dynamic viscosity μ and it may be assumed that the velocity gradient is linear at all radii.

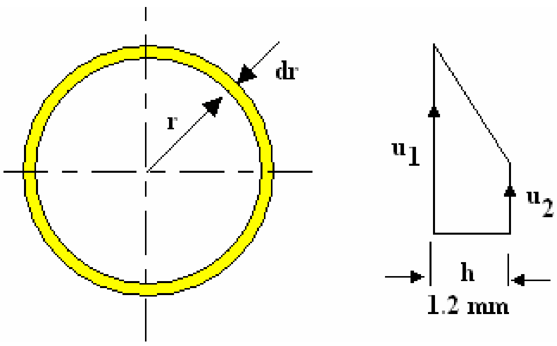


Show that the Torque at the input shaft is given by $T = \dfrac{\pi D^4 \mu(\omega_1 - \omega_2)}{32h}$

The input shaft rotates at 900 rev/min and transmits 500W of power. Calculate the output speed, torque and power. (747 rev/min, 5.3 Nm and 414 W)

Show by application of max/min theory that the output speed is half the input speed when maximum output power is obtained.

SOLUTION



Assume the velocity varies linearly from u_1 to u_2 over the gap at any radius. Gap is h = 1.2 mm

$T = \mu\, du/dy = \mu\,(u_1 - u_2)/h$

For an elementary ring radius r and width dr the shear force is

Force = $\tau\, dA = \tau\, 2\pi r\, dr$

$$dF = \mu \frac{u_1 - u_2}{h} x 2\pi r\ dr$$

Torque due to this force is $\quad dT = rdF = \mu \dfrac{u_1 - u_2}{h} x\ 2\ \pi\ r^2\ dr$

Substitute u = ωr $\quad dT = rdF = \mu \dfrac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi\ r^3\ dr$

Integrate $\quad T = \mu \dfrac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi \displaystyle\int_0^R r^3\ dr\ = \mu \dfrac{(\omega_1 - \omega_2)}{h} x\ 2\ \pi \dfrac{R^4}{4}$

Rearrange and substitute R = D/2 $\quad T = \mu \dfrac{(\omega_1 - \omega_2)}{h} x\ \pi \dfrac{D^4}{32}$

Put D = 0.3 m, $\mu = 0.5$ N s/m^2, h = 0.012 m $\quad T = 0.5 \dfrac{(\omega_1 - \omega_2)}{0.012} x\ \pi \dfrac{0.3^4}{32} = 0.33(\omega_1 - \omega_2)$

N = 900 rev/min P = 500 W Power = 2πNT/60 $\quad T = \dfrac{60P}{2\pi\ N} = \dfrac{60 \text{ x } 500}{2\pi \text{ x } 900} = 5.305$ Nm

The torque input and output must be the same. $\omega_1 = 2\pi N_1/60 = 94.25$ rad/s

$5.305 = 0.33(94.25_1 - \omega_2)$ hence $\omega_2 = 78.22$ rad/s and $N_2 = 747$ rev/min

$P_2 = 2\pi N_2 T/60 = \omega_2 T = 78.22 \text{ x } 5.305 = 414$ W (Power out)

For maximum power output $dp_2/d\omega_2 = 0$ $\quad P_2 = \omega_2 T = 0.33(\omega_1\omega_2 - \omega_2^2)$

Differentiate $\quad \dfrac{dP_2}{d\omega_2} = 0.33(\omega_1 - 2\omega_2)$

Equate to zero and it follows that for maximum power output $\omega_1 = 2\ \omega_2$

And it follows $N_1 = 2\ N_2$ so $N_2 = 450$ rev/min

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