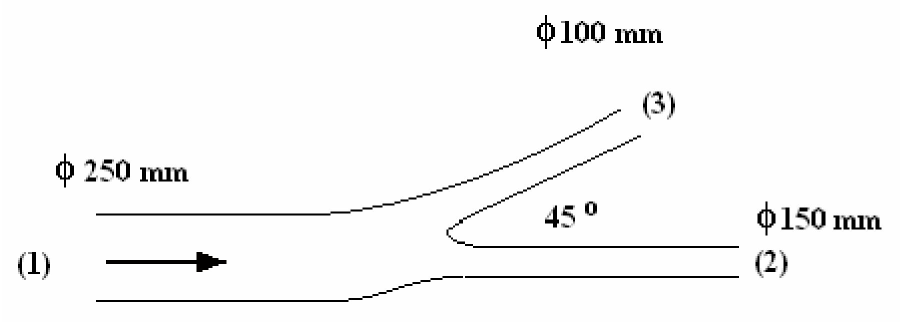
1 (a) State the conditions under which the Bernoulli equation is applicable.

(b) Water of density 1000 kg/m^3 is flowing into inlet (1) of the pipe-junction shown in the diagram. at a steady flow rate of 0.22 m^3/s. The volume of water in the junction is 0.016 m^3. The centre of the outlet (3) is situated 600 mm vertically above the main horizontal pipe running between (1) and (2). The water pressure at (1) is 230 kN/m^2 and at (2) is 200 kN/m^2; energy losses in the flow are considered negligible. Determine .

(i) the water pressure at (3).

(ii) the flows leaving the junction at (2) and (3).

(iii) the magnitude and direction of the force acting on the junction as a result of the flow.



D_1= 0.25m D_2= 0.15m D_3= 0.1m Q1= 0.22 m^3/s
p_1=230 x 10^3 N/m^2 p_2=200 x 10^3 N/m^2

$$A_1 = \frac{\pi D_1^2}{4} = 0.0491\,m^2 \quad A_2 = \frac{\pi D_2^2}{4} = 0.00177\,m^2 \quad A_3 = \frac{\pi D_3^2}{4} = 0.007854\,m^2$$

$$u_1 = \frac{Q_1}{A_1} = 4.482\ m/s$$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (2)

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$230\,x\,10^3 + 1000\frac{4.482^2}{2} = 200\,x\,10^3 + 1000\frac{u_2^2}{2}$$

$$u_2 = 8.95\ m/s$$

FLOW RATE $Q_2 = A_2 u_2 = 0.158\,m^3/s$

CONSERVATION OF MASS $Q_3 = Q_1 - Q_2 = 0.22 - 0.158 = 0.616\,m^3/s \quad u_3 = \frac{Q_3}{A_3} = 7.847\ m/s$

CONSERVATION OF ENERGY OF A STREAMLINE FROM (1) TO (3)

$$p_1 + \rho\frac{u_1^2}{2} = p_3 + \rho\frac{u_3^2}{2} + \rho g z_3$$

$$230\,x\,10^3 + 1000\frac{4.482^2}{2} = p_3 + 1000\frac{7.847^2}{2} + 1000\,x\,9.81\,x\,0.6$$

$$p_3 = 203.4\,x\,10^3\ N/m^2$$

FORCES

Force at (1) $F_1 = m_1 u_1 + A_1 p_1 \rightarrow$

$F_1 = 11390$ N

Force at (2) $F_2 = m_2 u_2 + A_2 p_2 \leftarrow$

$F_2 = 4960$ N

Force at (3) $F_3 = m_3 u_3 + A_3 p_3$ at 45^o

F3 = 2080 N

Horizontal component = $2080 \cos 45^o = 1470$ N $\leftarrow$

Vertical component = $2080 \sin 45^o = 1470$ N $\downarrow$

Total horizontal force = 11390 – 4960 – 1470 = 4960 N $\rightarrow$

Total vertical force = 1470 N $\downarrow$

RESULTANT FORCE = $\{49600^2 + 1470^2\}^{1/2} = 51731$ N

Angle = $\tan^{-1} 1470/4960 = 16.5^o$