FLUID MECHANICS D209 Q4 1996

(a) For laminar flow in a circular pipe, derive from first principles the following equation relating the head loss h_f, the pipe diameter d, the pipe length L, the mean velocity of flow u, the fluid density ρ, the fluid dynamic viscosity μ and the acceleration due to gravity g.

$$h_f = \frac{32\,\mu\,u\,L}{\rho g D^2}$$

(b) A 20 mm diameter, 5 m long pipe conveys oil of dynamic viscosity 1.20 N s/m^2 and density 900 kg/m^3 at a mean velocity of 0.30 m/s.
Show that the flow condition is laminar and determine

(i) The head loss
(ii) The centre line velocity
(iii) The radial location at which the velocity is equal to the mean velocity.

a) $C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$ Rearranging equation to make Δp the subject

$\Delta p = \frac{4C_f L\rho u_m^2}{2D}$ This is often expressed as a friction head h_f

$$h_f = \frac{\Delta p}{\rho g} = \frac{4C_f L u_m^2}{2gD}$$

This is the Darcy formula. In the case of laminar flow, Darcy's and Poiseuille's equations must give the same result so equating them gives

$$h_f = \frac{4C_f L u_m^2}{2gD} = \frac{32\mu 2\mu_m}{\rho g D^2}$$

b) Re = ρuD/μ = 900 x 0.3 x 0.2/1.2 = 4.5 and since this is much smaller than 2000 it must be laminar.

$$h_f = \frac{32\,\mu\,L\,u_m}{\rho g D^2} = \frac{32 \times 1.2 \times 0.3 \times 5}{900 \times 9.81 \times 0.02^2} = 16.3\,\text{m}$$

$\Delta p = \rho g h_f$ = 900x 9.81 x 16.3 = 144 kN/m^2

Centre Line velocity is twice the mean for laminar flow so u = 2 x 0.3 =0.6 m/s

At any other radius it is given by

$$u = \frac{\Delta p\left(R^2 - r^2\right)}{4\mu L} \qquad 0.3 = \frac{144000\left(0.01^2 - r^2\right)}{4 \times 1.2 \times 5}$$

$50 \times 10^{-6} = 0.01^2 - r^2$

r = 0.00707 m or 7.07 mm