(a) Show by dimensional analysis that the drag D on a sphere diameter d moving constant velocity v through a stagnant fluid of density ρ and dynamic viscosity μ may be expressed as

$$D = \frac{\mu^2}{\rho}\varphi\left(\frac{\rho v d}{\mu}\right)$$

Demonstrate that Stokes' law D =3πμvd is consistent with this expression and state the circumstances that under which Stokes' law is valid.

(b) A sample of particulate material of relative density 2.65 settled 2500 mm in still water in 4.7 s. Assuming that the flow regime corresponds to flow past a sphere, for which a drag coefficient C_D = 0.44 applies, calculate the equivalent spherical diameter of the particles.

Demonstrate that, in fact, the constant drag coefficient flow regime is applicable.

(c) Water flowing at 10 m^3/s carries particulate material identical to that in (b). The material is carried in low concentration, but it is required to remove all the material from the flow. For this purpose a 10 m long settling channel of rectangular cross-section, and in which the particles must reach the bed within the length of the channel, is to be designed. The flow velocity must not exceed 0.50 m/s and it may be assumed that it does not vary with depth.
Determine the minimum appropriate width and depth for the settling channel.

D = function (d v ρ μ) = $K\ d^a\ v^b\ \rho^c\ \mu^d$

First write out the MLT dimensions.

$[D] = ML^1T^{-2}$

$[d] = L$ $\qquad ML^1T^{-2} = L^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^d$

$[v] = LT^{-1}$ $\qquad ML^1T^{-2} = L^{a+b-3c-d}\ M^{c+d}\ T^{-b-d}$

$[\rho] = ML^{-3}$

$[\mu] = ML^{-1}T^{-1}$

Viscosity is the quantity which causes viscous friction so the index associated with it (d) is the one to identify. We will resolve a,b and c in terms of d as before.

TIME	-2 = -b - d	hence b = 2 - d is as far as we can resolve b
MASS	1 = c + d	hence c = 1 - d
LENGTH	1= a + b - 3c - d	

1 = a +(2 -d) -3(1-d) - d hence a = 2-d

Next put these back into the original formula. D = $K\ d^{2-d}\ v^{2-d}\ \rho^{1-d}\ \mu^d$

Next group the quantities with same power together as follows :

$$D = K\{\rho v^2 d^2\}\ \{\mu\ \rho^{-1} v^{-1} d^{-1}\}^d$$

$$\frac{D}{\rho v^2 d^2} = \varphi\left(\frac{\rho v d}{\mu}\right)$$

How this becomes $D = \frac{\mu^2}{\rho}\varphi\left(\frac{\rho v d}{\mu}\right)$ is not known.

STOKES LAW

D =3πμvd if we multiply by $\frac{\rho\mu}{\rho\mu}$

$$D = 3\pi\ \mu\ v\ d\frac{\rho\mu}{\rho\mu} = 3\pi\frac{\mu^2}{\rho}\frac{\rho v\ d}{\mu} = \frac{\mu^2}{\rho}\varphi\frac{\rho v\ d}{\mu}$$ so this is consistent.

Stokes flow applies to $R_e < 0.2$

(b) $C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2}$ u_t = 2.5/4.7 = 0.532 m/s ρ_f = 2650 kg/m^3

$$C_D = \frac{8d\ 9.81(2650 - 997)}{6 \text{ x } 997 \text{ x } 0.532^2} = 76.62d$$

0.44 = 76.62d hence d = 0.00574 m or 5.74 mm

Re =ρud/μ = 3420

Consistent with Newton flow since C_D = 0.44 Re is between 500 and 100000

(c) Q = Au A = 10/0.5 = 20 m^2

A = wD Time to cross the tank t = 10/0.5 = 20 s

Time to fall to bottom must be more.

Terminal velocity = 0.532 m/s

D = 0.532 x 20 = 10.64 m

W = 20/10.64 = 1.88 m