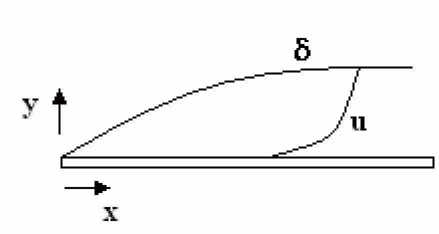
The velocity profile for flow over a flat plate with negligible pressure gradient in the flow direction may be approximated to $u/u_1=(3/2)(\eta) - (1/2)(\eta)^3$

$\eta = y/\delta$ and u is the velocity at a distance y from the plate and u_1 is the mainstream velocity. δ is the boundary layer thickness.

Discuss whether this profile satisfies appropriate boundary conditions.

Show the outline form of the derivation $C_f = 0.646 (R_{ex})^{-0.5}$ and evaluate the constant A.

SOLUTION



y = 0 u = 0 this is satisfied.
$y = \delta$ $u = u_1$ $\eta = 1$
$u/u_1=1 = (3/2)(1) - (1/2)(1)^3 = 1$ this is satisfied

$$\frac{du}{dy} = u\left\{\frac{3}{2}x\frac{1}{\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right\}$$

$$y = \delta \qquad \frac{du}{dy} = u\left\{\frac{3}{2\delta} - \frac{3}{2\delta}\right\} = 0 \text{ this is satisfied.}$$

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right]dy$$ leads to the solution $\theta = 39\delta/280$

Without proof that $C_f = 2\, d\theta/dx$ leads to $C_f = (78/280)d\delta/dx = 2\tau_o/\rho u^2$

$$\tau_o = \mu\,(du/dy)_{y=0} = u\mu\left\{\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right\} = \left\{\frac{3\mu u}{2\delta}\right\}$$

$$\frac{78}{280}\frac{d\delta}{dx} = \frac{2}{\rho u^2}\left\{\frac{3\mu u}{2\delta}\right\}$$

$$\delta\, d\delta = \frac{280}{78}\left\{\frac{3\mu}{\rho u}\right\}dx$$

$$\frac{\delta^2}{2} + C = \frac{280}{78}\left\{\frac{3\mu x}{\rho u}\right\}$$ but at $\delta = 0$, $x = 0$ so $C = 0$

$$\delta = \sqrt{21.538\left\{\frac{\mu x^2}{\rho u x}\right\}} = 4.64xR_{ex}^{-1/2}$$

$\delta/x = 4.64\, R_{ex}^{-1/2}$

$$C_f = \frac{2\tau_o}{\rho u^2} = 2\frac{3\mu u}{2\delta\rho u^2} = \frac{3\mu}{\delta\rho u}$$

$$C_f = \frac{3\mu x}{\delta\rho u x} = \frac{3\mu}{\rho u x 4.64R_{ex}^{-1/2}} = \frac{0.646}{R_{ex}R_{ex}^{-1/2}} = 0.646R_{ex}^{-1/2}$$