Q9 D203 FLUID MECHANICS 2003

PART A

Conducting the usual force balance on a cylindrical core of radius r we have:

$\tau\ 2\pi r\, dL = \pi r^2\, dp$ but in this case $\tau = C(-du/dr)^2$ substitute for τ

$C(-du/dr)^2\, 2\pi r\, dL = \pi r^2\, dp$

$C(-du/dr)^2 = r\, dp/dL$

$$\left(-\frac{du}{dr}\right)^2 = \frac{r}{2C}\frac{dp}{dL}$$

$$\frac{du}{dr} = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} r^{\frac{1}{2}}$$

$$du = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} r^{\frac{1}{2}} dr$$

$$\int_0^u du = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} \int_R^r r^{\frac{1}{2}} dr$$

$$u = -\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}} \frac{2}{3}\left[r^{\frac{3}{2}}\right]_R^r$$

$$u = -\frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[r^{\frac{3}{2}} - R^{\frac{3}{2}}\right] = \frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[R^{\frac{3}{2}} - r^{\frac{3}{2}}\right]$$ At the centre line r = 0 $u = U_o$

$$U_o = \frac{2}{3}\left(\frac{1}{2C}\frac{dp}{dL}\right)^{\frac{1}{2}}\left[R^{\frac{3}{2}}\right]$$ the ratio u/U_o is hence.

$$\frac{u}{U_o} = \frac{R^{\frac{3}{2}} - r^{\frac{3}{2}}}{R^{\frac{3}{2}}} = 1 - \left(\frac{r}{R}\right)^{\frac{3}{2}}$$

Note the question omitted the C and the minus sign results from putting –dp/dL

PART B

Consider the flow rate through a thin annular ring radius r and width dr.

dQ = u x 2πr dr

$$dQ = U_o \left\{ 1 - \left(\frac{r}{R} \right)^{\frac{3}{2}} \right\} x\ 2\pi\ rdr = 2\pi\ U_o \left\{ r - \left(\frac{r^{\frac{5}{2}}}{R^{\frac{3}{2}}} \right) \right\} dr$$

$$Q = 2\pi\ U_o \int_0^R r - \left(\frac{r^{\frac{5}{2}}}{R^{\frac{3}{2}}} \right) dr = 2\pi\ U_o \left[\frac{r^2}{2} - \left(\frac{2}{7} \frac{r^{\frac{7}{2}}}{R^{\frac{3}{2}}} \right) \right]_0^R$$

$$Q = 2\pi\ U_o \left[\frac{R^2}{2} - \left(\frac{2}{7} \frac{R^{\frac{7}{2}}}{R^{\frac{3}{2}}} \right) \right] = 2\pi\ U_o \left[\frac{R^2}{2} - \left(\frac{2R^2}{7} \right) \right]$$

$$Q = 2\pi\ U_o \frac{3}{14} R^2 = \frac{3\pi}{7} U_o R^2$$

PART C

D = 0.05 mm R = 0.025 dp/dL = 20 000 N/m^3 C=0.5 Ns^2/m^2

$$U_o = \frac{2}{3} \left(\frac{1}{2C} \frac{dp}{dL} \right)^{\frac{1}{2}} \left[R^{\frac{3}{2}} \right] = \frac{2}{3} \left(\frac{20000}{2\ x\ 0.5} \right)^{\frac{1}{2}} 0.025^{\frac{3}{2}} = 0.3726\ m/s$$

$$Q = \frac{3\pi}{7} U_o R^2 = \frac{3\pi}{7} x\ 0.3726\ x\ 0.025^2 = 0.000314\ m^3/s$$