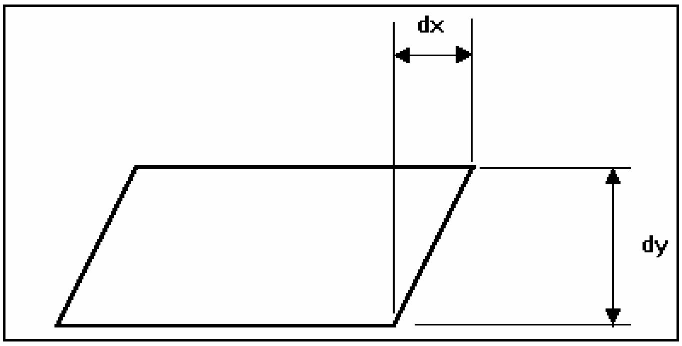
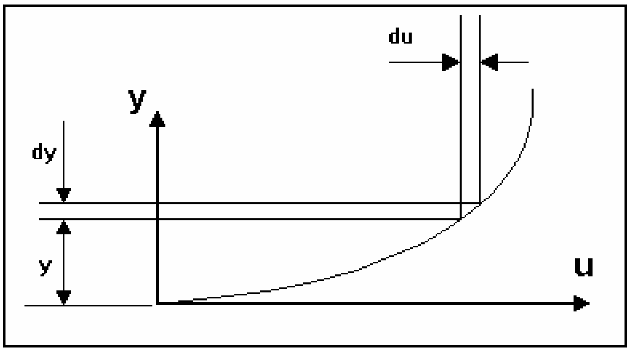
FLUID MECHANICS D203 SOLUTION Q5 2003

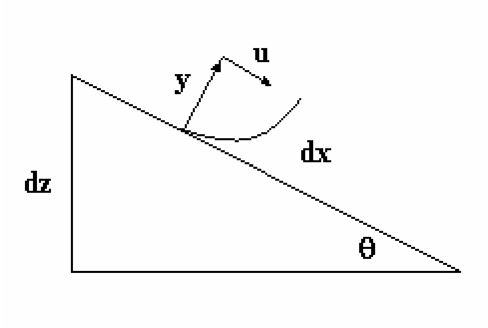
PART A

The normal equation for laminar flow is dp dy = - dτ dx  so dp/dx = - dτ/dy




and since $\tau = \mu du/dy$ for a Newtonian fluid $dp/dx = - \mu d^2u/dy^2$
In this case the flow is due to gravity only and pressure is related to height z by $p = \rho g z$
It follows that $dp = \rho g\, dz$




$dz/dx = \sin\theta$   hence $dp = \rho g\, dx \sin\theta$  and  $dp/dx = \rho g \sin\theta = - \mu d^2u/dy^2$

$g \sin\theta = - (\mu/\rho) d^2u/dy^2 = - (\nu) d^2u/dy^2$      $\mu/\rho = \nu$  the kinematic viscosity

**$n\, d^2u/dy^2 = - g \sin q$**

PART B

Integrate and    $du/dy = - (g/\nu)\, y \sin\theta + A$
Integrate again and    $u = - (g/\nu)(y^2/2) \sin\theta + Ay + B$   A and B  are constants of integration.

Boundary conditions
Put $du/dy = 0$ at $x = h$  so    $du/dy = 0 = - (g/\nu)\, h \sin\theta + A$    $A = (g/\nu)\, h \sin\theta$
Put $y = 0$ and $u = 0$ and it follows that $B = 0$

$u = - (g/\nu)(y^2/2) \sin\theta + \{(g/\nu)\, h \sin\theta\}\, y$
$u = (g/\nu) \sin\theta\{hy - y^2/2\}$

**$u = (g/2n) \sin q\{2hy - y^2\}$**

PART C

$\nu = 8 \times 10^{-5}\ m^2/s$  h = 0.005 m  Consider the flow through a small slit 1 m wide and width dy.

dQ = u  dy = u = $(g/2\nu) \sin\theta\{2hy - y^2\}$ dy

Integrate between y = 0 and y = h

$$Q = \frac{g}{2\nu}\sin\theta\left[\frac{2hy^2}{2} - \frac{y^3}{3}\right]_0^h = \frac{g}{2\nu}\sin\theta\left[h^3 - \frac{h^3}{3}\right] = \frac{g}{2\nu}\sin\theta\left[\frac{2h^3}{3}\right]$$

Evaluate and

$$Q = \frac{9.81}{2 \times 8 \times 10^{-5}}\sin 40\left[\frac{2 \times 0.005^3}{3}\right] = 0.003284\ m^3/s$$

Maximum velocity is at y = h so

$$U = \frac{g}{2\nu}\sin\theta\left(2h^2 - h^2\right) = \frac{g}{2\nu}\sin\theta\left(h^2\right) = \frac{9.81}{2 \times 8 \times 10^{-5}}\sin 40\left(0.005^2\right) = 0.985\ m/s$$