D203 FLUID MECHANICS SOLUTION Q4 2003

Comment – If anyone could do this question in the time allocated they would need to be a genius or have revised it so thoroughly ha hey could repeat it from memory. You also need to know that the stream function Ψ is taken as positive in the x direction and this is the opposite of most advanced books and that used n my tutorial. It follows that u = dΨ/dy and not - dΨ/dy

PART A

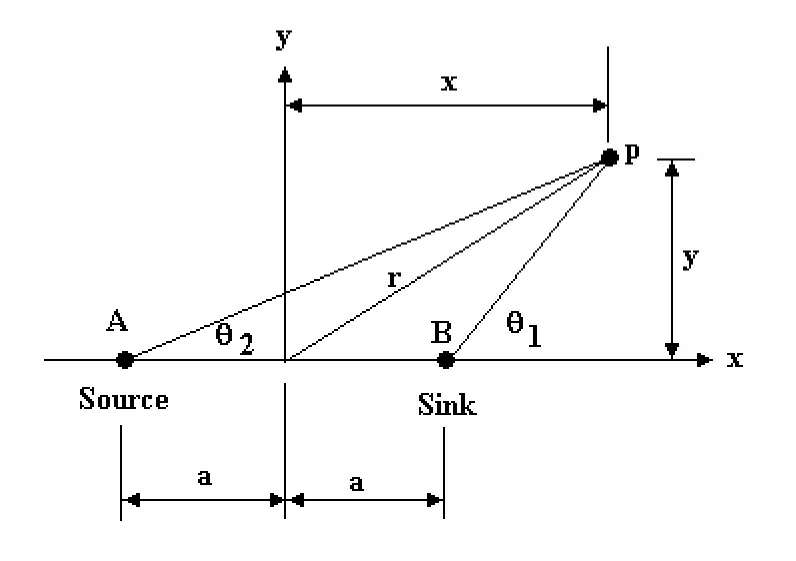
The combined stream function is found as follows.

Ψ_A= U y for a uniform flow U is the velocity of the uniform stream

$\Psi_B = \frac{m}{2p} q_2$ for the source $\Psi_C = -\frac{m}{2p} q_1$ for the sink

$$\Psi = \Psi_A + \Psi_B + \Psi_C = Uy + \frac{m}{2p}(q_2 - q_1)$$

Referring to the diagram for a source and sink placed on the x axis distance a either side :-



$$\tan q_1 = \frac{y}{x-a} \qquad \tan q_2 = \frac{y}{x+a}$$

$$\tan(q_2 - q_1) = \frac{\tan q_2 - \tan q_1}{1 + \tan q_2 \tan q_1}$$

$$\tan(q_2 - q_1) = \frac{\frac{y}{x+a} - \frac{y}{x-a}}{1 + \left(\frac{y}{x+a}\right)\left(\frac{y}{x-a}\right)}$$

$$\tan(q_2 - q_1) = \frac{y(x-a) - y(x+a)}{1 + \frac{y^2}{x^2 - a^2}}$$

$$\tan(q_2 - q_1) = \frac{y(x-a) - y(x+a)}{1 + \frac{y^2}{x^2 - a^2}}$$

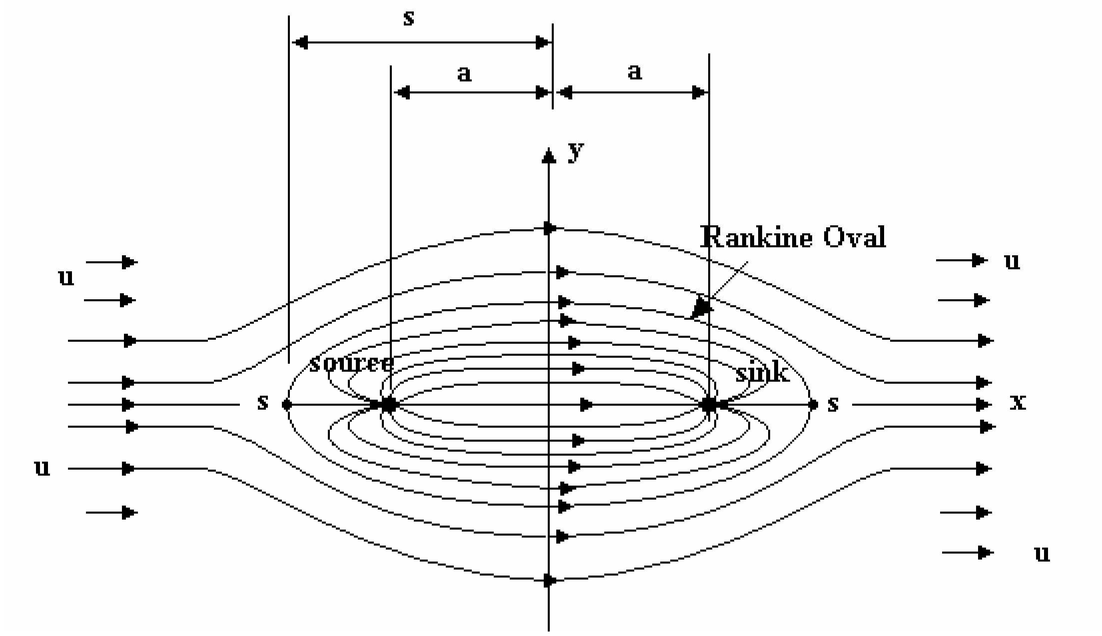
$$\tan(q_2 - q_1) = \frac{-2ay}{x^2 - a^2 + y^2}$$

$$-\tan(q_2 - q_1) = \frac{2ay}{x^2 - a^2 + y^2}$$

$$\Psi = Uy - \frac{m}{2p}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$$

PART B

$\Psi = Uy - \frac{m}{2p}\tan^{-1}\left(\frac{2ay}{x^2 + y^2 - a^2}\right)$ The stream pattern is like this:



The entire output of the source flows inside the Rankine Oval which is the zero stream line. There is no flux across this line. Putting Ψ = 0 and x = 0 gives:

$$0 = Uy - \frac{m}{2p}\tan^{-1}\left(\frac{2ay}{y^2 - a^2}\right)$$

At this point y is the half width of the oval h so change y to h and we get:

$$Uh = \frac{m}{2p}\tan^{-1}\left(\frac{2ay}{y^2 - a^2}\right)$$

$$\tan\left(\frac{2pUh}{m}\right) = \frac{2ah}{h^2 - a^2} \qquad h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2pUh}{m}\right)$$

<u>VELOCITY IN THE x DIRECTION</u>

The velocity in the x direction is given by $u = \frac{\partial \Psi}{\partial y}$

This is easier to solve by using $\Psi = Uy + \frac{m}{2p}(q_2 - q_1) = Uy + \frac{m}{2p}\left(\tan^{-1}\frac{y}{x+a} - \tan^{-1}\frac{y}{x-a}\right)$

$$u = \frac{\partial \Psi}{\partial y} = U + \frac{m}{2p}\left[\left\{\frac{1}{x+a}\right\}\left\{1 + \left(\frac{y}{x+a}\right)^2\right\}^{-1} - \left\{\frac{1}{x-a}\right\}\left\{1 + \left(\frac{y}{x-a}\right)^2\right\}^{-1}\right]$$

At the stagnation point, the velocity is zero, y = 0 and x = ±s hence:

$$u = 0 = U + \frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = U + \frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$u = 0 = U + \frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = U + \frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$U = -\frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right]$$

$$u = 0 = U + \frac{m}{2p}\left[\left\{\frac{1}{s+a}\right\}\left\{1 + \left(\frac{0}{s+a}\right)^2\right\}^{-1} - \left\{\frac{1}{s-a}\right\}\left\{1 + \left(\frac{0}{s-a}\right)^2\right\}^{-1}\right]$$

$$\frac{2Up}{m} = -\left[\left\{\frac{1}{s+a}\right\} - \left\{\frac{1}{s-a}\right\}\right] = \left[\frac{(s-a)-(s+a)}{(s+a)(s-a)}\right] = \frac{-2a}{s^2 - a^2}$$

$$s^2 - a^2 = \frac{2ma}{2Up} = \frac{ma}{Up} \qquad s^2 - a^2 = \frac{2ma}{2Up} = \frac{ma}{Up}$$

$$s^2 = \frac{ma}{Up} + a^2 s^2 = \frac{ma}{Up} + a^2$$

$$s = \sqrt{a^2 + \frac{ma}{Up}}$$

PART C

2a = 0.1562 a = 0.0781 u = 3 m/s h = 0.05

$$h = \frac{h^2 - a^2}{2a}\tan\left(\frac{2puh}{m}\right) \text{ put } h = 0.05$$

$$0.05 = \frac{0.05^2 - 0.0781^2}{2 \times 0.0781}\tan\left(\frac{2p \times 3 \times 0.05}{m}\right)$$

$$-2.1696 = \tan\left(\frac{0.9425}{m}\right) \text{ remember to work in radian mode}$$

$$\pm 1.1389 = \frac{0.9425}{m} \quad m = \pm 1.208$$

Length of Rankine Oval = 2s = 0.2 so s = 0.1 s = 0.1 = $\sqrt{\{(ma/\pi u) + a^2\}}$
0.01 = ma/πu) + a^2 (1.208 x 0.0781)/(π x 3) = 0.01 As both give 0.01 the data is correct.

VELOCITY
If anyone can show me how to solve this part I would be grateful.