D203 FLUID MECHANICS QUESTION 1 2003

PART A

$a = f(K, \rho) = C K^a \rho^b$ dimensions are:
$[a] = [m/s] = LT^{-1}$
$[K] = [N/m^2] = [(kg\ m/s^2)(1/m^2)] = ML^{-1}T^{-2}$
$[\rho] = [kg/m^3] = ML^{-3}$ Hence

$M^0LT^{-1} = C\ (ML^{-1}T^{-2})^a(ML^{-3})^b = M^aL^{-a}T^{-2a}M^bL^{-3b}$
Equate powers

Time	$-1 = -2a$	$a = ½$
Mass	$0 = a + b$	$b = -½$ Substitute back $\mathbf{a = C K^{1/2} \rho^{-1/2} = C\sqrt{(K/\rho)}}$

PART B

$R = \text{function}\ (l\ v\ \rho\ \mu\ K\ g)$

There are 7 quantities and there will be 3 basic dimensions ML and T. This means that there will be 4 dimensionless numbers Π_1, Π_2, Π_3 and Π_4. These numbers are found by choosing four prime quantities (R, μ, K and g).

Π_1 is the group formed between R and l v ρ
Π_2 is the group formed between μ and l v ρ
Π_3 is the group formed between K and l v ρ
Π_4 is the group formed between g and l v ρ

The first is formed by combining R with ρ,v and l
$R = \Pi_1\ l^a v^b \rho^c$
$MLT^{-2} = \Pi_1\ (L)^a (LT^{-1})^b (ML^{-3})^c$

Time	$\mathbf{-2 = -b}$	$\mathbf{b = 2}$
Mass		$\mathbf{c = 1}$
Length	$\mathbf{1 = a + b - 3c}$	
	$1 = a + 2 - 3$	$\mathbf{a = 2}$

$R = \Pi_1\ l^2 v^2 \rho^1$ $\qquad \Pi_1 = \dfrac{R}{\rho v^2 l^2}$ and this is the Newton number.

The second is formed between μ and ρ,v and l.
$\mu = \Pi_2\ l^a v^b \rho^c$
$M^1L^{-1}T^{-1} = \Pi_2\ (L)^a (LT^{-1})^b (ML^{-3})^c$

Time	$-1 = -b$	$b = 1$
Mass		$\mathbf{c = 1}$
Length	$-1 = a + b - 3c$	
	$-1 = a + 1 - 3$	$\mathbf{a = 1}$

$\mu = \Pi_2\ l^1 v^1 \rho^1$ $\qquad \Pi_2 = \dfrac{\mu}{lv\rho}$ and lvρ/μ is the Reynolds number

The third group is formed between K and l v ρ

$K = \Pi_3\ l^a v^b \rho^c$
$ML^{-1}T^{-2} = \Pi_3\ L^a (LT^{-1})^b (ML^{-3})^c$
$ML^{-1}T^{-2} = \Pi_3\ L^{a+b-3c} M^c T^{-b}$

Time -2 = -b **b = 2**
Mass **c = 1**
Length -1 = a + b -3c
-1 = a + 2 – 3 **a = 0**

$K = \Pi_3\ l^o\ v^2\ \rho^1$

$$\Pi_3 = \frac{K}{\rho v^2}$$

It was shown earlier that the speed of sound in an elastic medium is given by the following formula. $a = C(k/\rho)^{1/2}$

It follows that $(k/\rho) = a^2$ and so $\Pi_3 = C^2(a/v)^2 = C^2/M^2$

The fourth group is formed between g and l v ρ

$g = \Pi_4\ l^a\ v^b\ \rho^c$
$LT^{-2} = \Pi_4\ L^a\ (LT^{-1})^b\ (ML^{-3})^c$
$M^0L^1T^{-2} = \Pi_4\ L^{a+b-3c}\ M^c\ T^{-b}$

Time -2 = -b **b = 2**
Mass **c = 0**
Length -1 = a + b -3c
1 = a + 2 **a = -1**

$g = \Pi_4\ l^{-1}\ v^2$ $\qquad \Pi_4 = \frac{gl}{v^2}$

The Froude Number is defined as $Fr = v/\sqrt{(gl)}$ so $\Pi_4 = 1/Fr^2$

Putting it all together we have $\Pi_1 = \frac{R}{\rho v^2 l^2}$ **= f(Π_2, Π_3, Π_4) = f(Re)(M)(Fr)**

All powers and constants are implied in the function sign.

PART C

l = 150 m

v = 30 km/h = 8.333 m/s

For dynamic similarity of the Froude number only

$Fr = v/\sqrt{gl} = 8.333/\sqrt{(9.81 \times 150)} = 0.217$

For the model we must have the same Froude number. Lm = 150/40 = 3.75

$Fr = 0.217 = v_m/\sqrt{gl_m} = v_m/\sqrt{(9.81 \times 3.75)}$ hence v_m = 1.318 m/s or 4.743 km/h