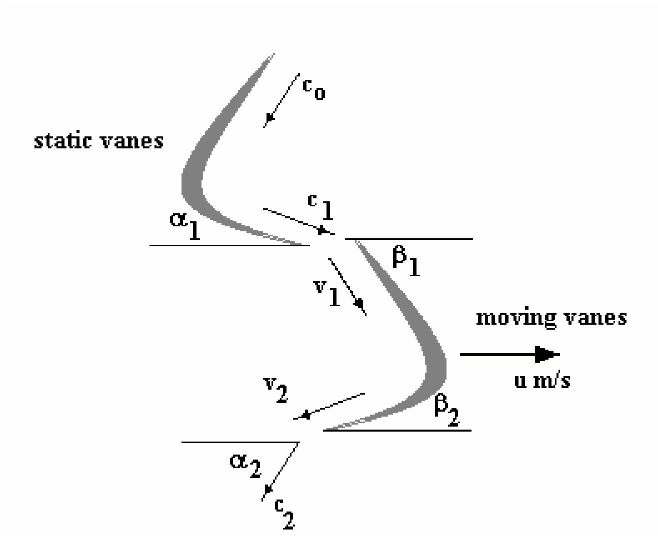
7 (a) Sketch the velocity diagram for the mean-diameter stator and rotor sections of a stage of an axial-flow reaction turbine. Assume equal inlet and outlet velocities to the stage and constant axial flow velocity. Indicate on the diagram all the angles which the absolute and relative velocity vectors make with the tangential, which is the whirl, direction.

(b) The degree of reaction **DR** is the ratio of the rotor enthalpy drop to the stage enthalpy drop. Prove that

$$DR = \frac{V_a}{2U}\left(\cot\beta_2 - \cot\beta_1\right)$$

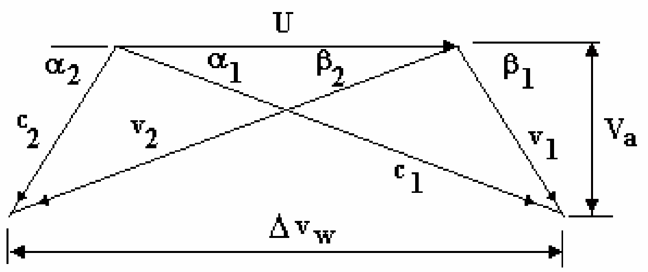
where $\frac{V_a}{U}$ is the ratio of the axial flow velocity to the rotor blade velocity, and β_1, and β_2, are the rotor blade inlet and outlet angles respectively.

(c) The mean-diameter section of a stage with **DR** = 0.5, has a blade velocity of 150 m/s and an axial gas velocity of 120 m/s. If the temperature drop across the stage is 25 °C and the specific heat capacity at constant pressure C_P is 1.0 kJ/kgK, calculate all stator and rotor angles.



The stationary vane makes an angle α_1 with the direction of rotation. The moving vane has an angle β_1 at inlet and β_2 at outlet. c is the absolute velocity of the steam and v is the relative velocity. The velocity diagram is as shown if the absolute velocity entering the stationary vanes is the same as the absolute velocity c_2 at exit from the moving rotor. In this event it follows that $\beta_1 = \alpha_2$ and $\beta_2 = \alpha_1$.

U = blade velocity. V_a = Axial velocity.
Δv_w = change in velocity in whirl direction.
Enthalpy at entry to stage = h_o
Enthalpy at exit from stage = h_2
Change in enthalpy = work given to the rotor
$h_o - h_2 = U\,\Delta v_w$ $\qquad \Delta v_w = V_a(\cot\beta_1 + \cot\beta_2)$
h_1 = enthalpy at entry to the rotor. Change in enthalpy over the rotor = change in KE over the rotor



$$h_1 - h_2 = \frac{v_2^2 - v_1^2}{2}$$

$v_2 = V_a \operatorname{cosec}\beta_2 \qquad v_1 = V_a \operatorname{cosec}\beta_1$

$$h_1 - h_2 = V_a^2\left\{\frac{\left(\operatorname{cosec}^2\beta_2 - \operatorname{cosec}^2\beta_1\right)}{2}\right\}$$ but since

$(\operatorname{cosec}\beta)^2 = (\cot\beta)^2 + 1$

$$h_1 - h_2 = V_a^2\left\{\frac{\left(\cot^2\beta_2 - \cot^2\beta_1\right)}{2}\right\}$$

$$h_1 - h_2 = \frac{V_a^2}{2}\left(\cot\beta_2 + \cot\beta_1\right)\left(\cot\beta_2 - \cot\beta_1\right)$$

$$DR = \frac{h_1 - h_2}{h_o - h_2} = \frac{V_a^2}{2}\left\{\frac{\left(\cot\beta_2 + \cot\beta_1\right)\left(\cot\beta_2 - \cot\beta_1\right)}{UV_a\left(\cot\beta_2 + \cot\beta_1\right)}\right\} = \frac{V_a}{2U}\left(\cot\beta_2 - \cot\beta_1\right)$$

DR = 0.5 $\quad$ U = 150 m/s $\quad$ V_a = 120 m/s $\quad$ ΔT = 25 K $\quad$ C_P is 1.0 kJ/kgK

$C_p\ \Delta T$ = change in enthalpy over the stage = $U\ \Delta v_w$
$C_p\ \Delta T$ = change in enthalpy over the stage = Δv_w
$\Delta v_w = C_p\ \Delta T/U = 45\ 000/150 = 300$ m/s
$\Delta v_w = V_a(\cot\beta_1 + \cot\beta_2)$ $300 = 120(\cot\beta_1 + \cot\beta_2)$ $\cot\beta_1 = 2.5 - \cot\beta_2$

$$DR = 0.5 = \frac{V_a}{2U}\left(\cot\beta_2 - \cot\beta_1\right)$$

$$0.5 = \frac{120}{2x\,150}\left(\cot\beta_2 - \cot\beta_1\right)$$

$$1.25 = \left(\cot\beta_2 - \cot\beta_1\right)$$

$$\cot\beta_1 = \cot\beta_2 - 1.25$$

$\cot\beta_1 = 2.5 - \cot\beta_2 = \cot\beta_2 - 1.25$
$2\cot\beta_2 = 3.75$ $\cot\beta_2 = 1.875$ $\tan\beta_2 = 0.5333$ $\beta_2 = 28^o = \alpha_1$
$\cot\beta_1 = 2.5 - \cot\beta_2 = 0.625$ $\tan\beta_1 = 1.6$ $\beta_2 = 58^o = \alpha_2$