6 A single-shaft gas-turbine jet engine is used as the propulsion unit on a small aircraft. The aircraft is flying at a velocity of 200 m/s at sea level where atmospheric pressure p is 1 bar and temperature T is 293 K. The pressure ratio over the compressor is 30. The compressor is adiabatic with an isentropic efficiency of 85%. After combustion, the hot gases enter the turbine with a temperature of 1200 K and expand adiabatically through the turbine. The turbine has an isentropic efficiency of 90% and it generates just sufficient power to drive the compressor. Finally the gases expand reversibly and adiabatically through a convergent propulsion nozzle, the outlet of which is choked.

(a) Determine the pressures at turbine and nozzle exits, the mass flow rate and the thrust developed if the nozzle has an exit area of 0.15 m^2.

(b) Also determine the power being generated to propel the aircraft.

Assume that the engine intake is isentropic, the working fluid throughout the engine is air with a gas constant R of 0.287 kJ/kgK, a specific heat capacity at constant pressure C_P of 1.0 kJ/kgK and an adiabatic constant γ of 1.4. Further assume that air is a perfect gas, and neglect all mechanical losses.

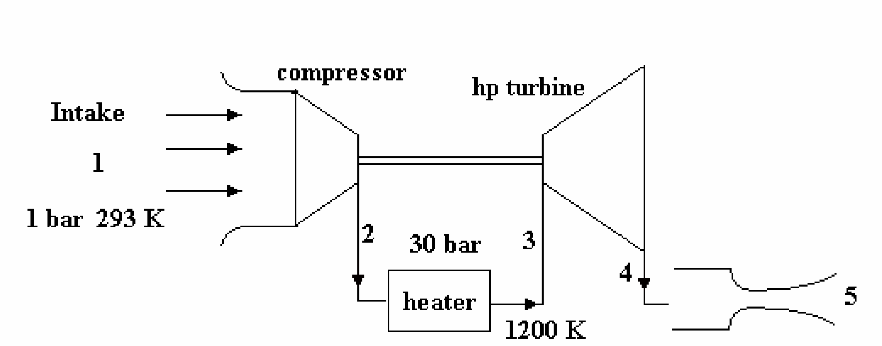
The critical temperature ratio in an isentropic nozzle is $\frac{2}{\gamma+1}$ and the velocity of sound is $\frac{\gamma p}{\rho}$

Where ρ is density.

The stagnation and static pressures p_o and p respectively are linked to the Mach number *M* by

$$\frac{p}{p_o} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{2}{\gamma - 1}}$$

(c) Show that an aircraft velocity of 200 m/s has an effect on the engine cycle.



COMPRESSOR

$$T_o = T_1 + \frac{u_1^2}{2c_p} = 293 + \frac{200^2}{2000} = 313\,K$$

$$T_2' = T_o (r_p)^{\frac{\gamma-1}{\gamma}} = 313\,x\,30^{0.2857} = 827\,K$$

$$\eta_i = 0.85 = \frac{827 - 313}{T_2 - 313} \qquad T_2 = 917.7\,K$$

Specific Power Input = $c_p\,\Delta T$ = 1 x (917.7- 313) = 604.7 kW

TURBINE

Power Out = Power In = 604.7 = $c_p\,\Delta T$ = 1 x (1200 – T_4) T_4 = 595.3 K

This is the actual temperature. Find the ideal temperature.

$\eta_i = 0.9 = \frac{1200 - 595.3}{1200 - T_4'}$ $T_4' = 528.1$ K

$\frac{T_4'}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$ $\frac{528.1}{1200} = \left(\frac{p_4}{30}\right)^{0.2857}$ $p_4 = 1.696$ bar

NOZZLE

$T_5 = T_4\left(\frac{2}{\gamma+1}\right) = 595.3 \text{ x } 0.833 = 496.1 \text{ K}$

$\frac{T_4}{T_5} = \frac{595.3}{496.1} = \left(\frac{p_4}{p_5}\right)^{0.2857}$ $1.2 = \left(\frac{1.696}{p_5}\right)^{0.2857}$ $p_5 = 0.896$ bar

or $p_5 = p_4\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 1.696\left(\frac{2}{2.4}\right)^{3.5} = 0.896 \text{ bar}$

This pressure is less than atmospheric so there must be shock waves????

Apply conservation of energy.
$c_p T_4 = c_p T_5 + u^2/2$

$1000 \text{ x } 595.3 = 1000 \text{ x } 496.1 + u^2/2$ $u = 951.5$ m/s

$V = A_2 u = 0.15 \text{ x } 951.5 = 142.725 \text{ m}^3/\text{s}$

m = pV/RT = $(0.896 \text{ x } 10^5 \text{ x } 142.725)/(287 \text{ x } 496.1)$ = kg/s

THRUST

$F_T = m(v - u) + A_2(p_2 - p_a) = 89.82\ (951.5 - 200) + 0.015\ (0.896 - 1.013) \text{ x } 10^5 = 67497 - 175.5$
$F_T = 67.32$ kN

NB I am not sure about the low pressure p_5. There must be some affect due to the pressure rise to atmospheric.

(b) POWER DEVELOPED

$P = F_T\ v = 67.32 \text{ x } 200 = 13464$ kW or 13.46 MW

(c) The entrance to the compressor must be a duct and a ram jet affect is achieved which affects the pressure rise and temperature rise over the compressor. I thought this was taken into account with the use of stagnation temperature and pressure so I don’t see the relevance of this part of the question. Anyone knowing the answer, please let me know.