4 (a) Show for helium that $\gamma = 5/3$ where γ is the adiabatic constant.

A closed-cycle single-shaft gas turbine plant using helium as the working fluid incorporates the following components in the given order: (a) a compressor, (b) a heater, (c) a two-stage turbine with reheater and (d) a cooler.

The maximum and minimum pressures and temperatures in the cycle are 40 bar and 700 °C, and 10 bar and 25 °C respectively, with reheat to 700 °C. The pressure in the reheater is optimum for maximum specific power (power per kg/s of gas flow).

The molar mass of helium is 4 kg/kmol and the molar heat capacity at constant volume for helium is $3/2\,\tilde{R}$ where $\tilde{R}$ = 8.3145 kJ/kmol K is the universal molar gas constant.

(b) Sketch the T-s diagram for the plant and indicate pressures and temperatures between the components if
(i) the reheater is used,
(ii) the reheater is by-passed.

(c) Calculate the ideal cycle efficiency and specific power for each case. Assume that there are no losses in the cycle.

(a) For Helium $\tilde{m} = 4$ (mol mass) $R = \tilde{R}/\tilde{m} = 8.3145/4 = 2.0786$ kJ/kg K

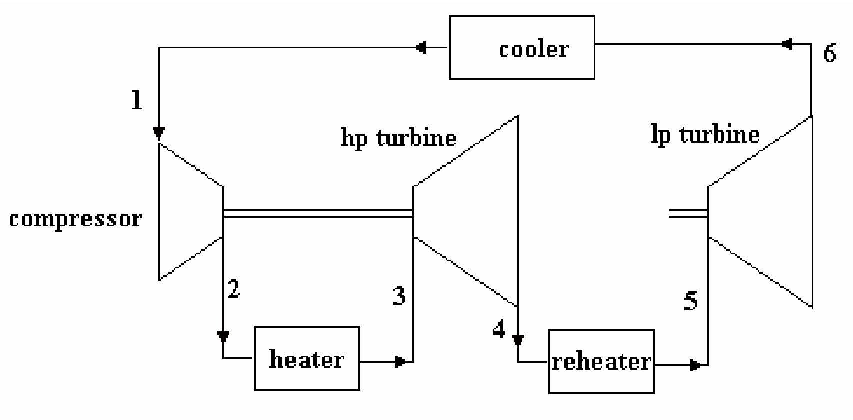
$$\tilde{c}_v = \frac{3\tilde{R}}{2}$$

$$\tilde{c}_p = \tilde{R} + \tilde{c}_v = \tilde{R} + \frac{3\tilde{R}}{2} = \frac{5\tilde{R}}{2}$$

$$c_v = \frac{3R}{2} = 3.1179 \text{ kJ/kg K}$$

$c_p = R + c_v = 5.1966$ kJ/kg K

$\gamma = c_p / c_v = 1.667$



$p_1 = 10$ bar $\theta_1 = 25^oC$ $T_1 = 298$ K

$p_2 = 40$ bar

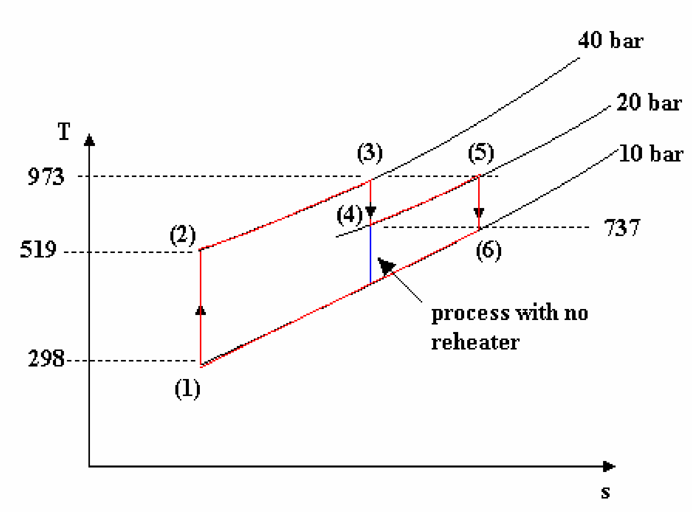
$p_3 = 40$ bar $\theta_3 = 700^oC$ $T_3 = 973$ K

For optimal turbine work $p_{4/5} = \sqrt{(40)(10)} = \sqrt{400} = 20$ bar $\theta_5 = 700^oC$ $T_5 = 973$ K

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = 298\left(\frac{40}{10}\right)^{1-\frac{1}{1.667}} = 518.9 \text{ K}$$

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{1-\frac{1}{\gamma}} = 973\left(\frac{20}{40}\right)^{1-\frac{1}{1.667}} = 737.4 \text{ K}$$

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{1-\frac{1}{\gamma}} = 973\left(\frac{10}{20}\right)^{1-\frac{1}{1.667}} = 737.4 \text{ K}$$



HEAT INPUT

$\Phi(in) = c_p(T_3 - T_2) + c_p (T_5 - T_4) = 5.1966(973 - 518.9) + 5.1966(973 - 734.7) = 3598.1$ kW

HEAT OUTPUT

$\Phi(out) = c_p(T_6 - T_1) = 5.1966(734.7 - 298) = 2269.4$ kW

Nett Power Out = 3598.1- 2269.4 = 1328.7 kW per kg/s of gas flow

Cycle efficiency η = P/Φ(in) = 1328.7/3598.1 = 0.369 or 36.9 % with reheater

With the reheater bypassed we have a standard Joule cycle.

$$\eta = 1 - r_p^{\frac{1}{\gamma}-1} = 1 - \left(\frac{40}{10}\right)^{\frac{1}{1.667}-1} = 0.426$$

HEAT INPUT

$\Phi(in) = c_p(T_3 - T_2) = 5.1966(973 - 518.9) = 2360$ kW

Nett Power Out = η x 2360 = 1005 kW per kg/s of gas flow