Q4 Sketch a pressure-volume diagram for the air-standard dual combustion cycle and describe the processes which occur in each part of the cycle.

In an air-standard dual combustion cycle, the temperature and pressure at the start of compression are 300 K and 1 bar respectively. The energy added in the cycle is $1600 \mathrm{~kJ} / \mathrm{kg}$, of which three-quarters is added at the constant volume and the remainder at the constant pressure parts of the cycle. The compression ratio is $20: 1$ and the compression and expansion strokes are polytropic with polytropic indices of $\mathrm{n}_{\mathrm{c}}=1.45$ and $\mathrm{n}_{\mathrm{e}}=1.35$ respectively.

Determine:
(a) the maximum pressure in the cycle
(b) the maximum temperature in the cycle
(c) the cycle efficiency
(d) the mean effective pressure.

Assume that $\mathrm{c}_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kgK}, \quad \mathrm{c}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kgK}$ and $\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kgK}$ and all remain constant throughout the cycle.

Comment - If the compression and expansion are not adiabatic, the cycle can not be an air standard cycle. The air standard efficiency formula cannot be used in this case.

The processes are as follows.
1-2 reversible (polytropic??) compression.
2-3 constant volume heating.
3-4 constant pressure heating.
4-1 reversible (polytropic??) expansion.
5-1 constant volume cooling.

$\mathrm{T}_{1}=300 \mathrm{~K} \quad \mathrm{p}_{1}=1 \mathrm{bar}$
$\mathrm{V}_{1} / \mathrm{V}_{2}=20$
$\mathrm{T}_{2}=300 \times 20^{\mathrm{n}-1}=300 \times 20^{1.45-1}=1155 \mathrm{~K}$
$\mathrm{p}_{2}=\mathrm{p}_{1} \mathrm{r}^{\mathrm{n}}=1 \times 20^{1.45}=77 \mathrm{bar}$
Heat Input at constant Volume is $0.75 \times 1600=1200 \mathrm{~kJ} / \mathrm{kg}$
$1200=\mathrm{mc}_{\mathrm{v}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=1 \times 0.718 \times\left(\mathrm{T}_{3}-1155\right) \quad \mathrm{T}_{3}=2826.3 \mathrm{~K}$
Heat Input at constant Pressure is $0.25 \times 1600=400 \mathrm{~kJ} / \mathrm{kg}$
$400=\mathrm{mc}_{\mathrm{p}}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)=1 \times 1.005 \times\left(\mathrm{T}_{4}-2826.3\right) \quad \mathrm{T}_{4}=3224.3 \mathrm{~K}$
This is the maximum temperature in the cycle.
$\mathrm{p}_{3}=\frac{\mathrm{p}_{1} \mathrm{~V}_{1} \mathrm{~T}_{3}}{\mathrm{~V}_{3} \mathrm{~T}_{1}}=\frac{1 \times 20 \times 2826.4}{1 \times 300}=188.42 \mathrm{bar}$
$\mathrm{p}_{4}=188.42$ bar This is the highest pressure in the cycle.
$\frac{\mathrm{V}_{1}}{\mathrm{~V}_{4}}=\frac{\mathrm{p}_{4} \mathrm{~T}_{1}}{\mathrm{p}_{1} \mathrm{~T}_{4}}=\frac{188.42 \times 300}{1 \times 3224.3}=17.53 / 1=\frac{\mathrm{V}_{5}}{\mathrm{~V}_{4}}$
$\mathrm{p}_{4} \mathrm{~V}_{4}{ }^{\mathrm{n}}=\mathrm{p}_{5} \mathrm{~V}_{5}{ }^{\mathrm{n}}$
$\mathrm{p}_{5}=\mathrm{p}_{4}\left(\frac{\mathrm{~V}_{4}}{\mathrm{~V}_{5}}\right)^{\mathrm{n}}=188.42\left(\frac{1}{17.53}\right)^{1.35}=3.95 \mathrm{bar}$
$\frac{\mathrm{p}_{5}}{\mathrm{~T}_{5}}=\frac{\mathrm{p}_{1}}{\mathrm{~T}_{1}} \quad \mathrm{~T}_{5}=\frac{\mathrm{p}_{5} \mathrm{~T}_{4}}{\mathrm{p}_{1}}=\frac{3.95 \times 300}{1}=1185 \mathrm{~K}$

The problem now is that because the work processes are polytropic, there is a heat transfer in these processes that makes it difficult to determine the heat rejected so we need to find the net work done. This involves a lot more work and I wonder if this is what the examiner intended?

Finding the true net work would require the work laws to be applied
COMPRESSION
$\mathrm{W}=\frac{\mathrm{p}_{2} \mathrm{~V}_{2}-\mathrm{p}_{1} \mathrm{~V}_{1}}{\mathrm{n}-1}=\frac{\mathrm{mR}\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)}{\mathrm{n}-1}=\frac{1 \times 287(300-1155)}{0.45}=-545.3 \times 10^{3} \mathrm{~J} / \mathrm{kg}$
EXPANSION
$\mathrm{W}=\frac{\mathrm{p}_{4} \mathrm{~V}_{4}-\mathrm{p}_{5} \mathrm{~V}_{5}}{\mathrm{n}-1}=\frac{\mathrm{mR}\left(\mathrm{T}_{4}-\mathrm{T}_{5}\right)}{\mathrm{n}-1}=\frac{1 \times 287(3224.3-1185)}{0.35}=1772.2 \times 10^{3} \mathrm{~J} / \mathrm{kg}$
There is also work in the constant pressure process
$\mathrm{W}=\mathrm{p}_{3}\left(\mathrm{~V}_{4}-\mathrm{V}_{3}\right)=\operatorname{mR}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)=1 \times 287(3224.3-2826.3)=114.2 \times 10^{3} \mathrm{~J} / \mathrm{kg}$
Net Work $=114.2+1772.2-545.3=1341.1 \mathrm{~kJ} / \mathrm{kg}$
$\eta=1341.1 / 1600=83.8 \%$
$\mathrm{V}_{1}=\mathrm{mRT}_{1} / \mathrm{p}_{1}=1 \times 287 \times 300 /\left(1 \times 10^{5}\right)=0.861 \mathrm{~m}^{3}($ based on 1 kg$)$
$\mathrm{V}_{2}=\mathrm{V}_{1} / 20=0.04305 \mathrm{~m}^{3}$ (based on 1 kg )
$\mathrm{MEP}=\mathrm{W}($ net $) /$ Swept Volume $=\mathrm{W}(\mathrm{net}) /\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)=1341.1 \times 10^{3} /(0.861-0.04305)=1.64 \times 10^{6} \mathrm{~Pa}$
This seems extremely high if anyone finds any errors in this work please contact admin@freestudy.co.uk

