QUESTION 3 2006

(a) The head loss in a pipe can be expressed in the form $h_f = KQ^2$. Two pipes having constants K_1 and K_2 are to be considered as a single equivalent pipe. Determine the value K_3 of this single pipe when the two are laid:

i. in series
ii. in parallel.

SOLUTION PART A

i. In series the flow is the same and total head loss is the sum of the two.

$h_{f1} = k_1Q^2 \qquad h_{f2} = k_2Q^2 \qquad h_{f1} + h_{f2} = k_3Q^2 = k_1Q^2 + k_2Q^2$

Hence $k_3 = k_1 + k_2$

ii. In parallel the friction heads are the same and the flows different.

$h_f = k_1Q_1^2 \qquad Q_1 = (h_f/k_1)^{1/2}$

$h_f = k_2Q_2^2 \qquad Q_2 = (h_f/k_2)^{1/2}$

$h_f = k_3(Q_1 + Q_2)^2$

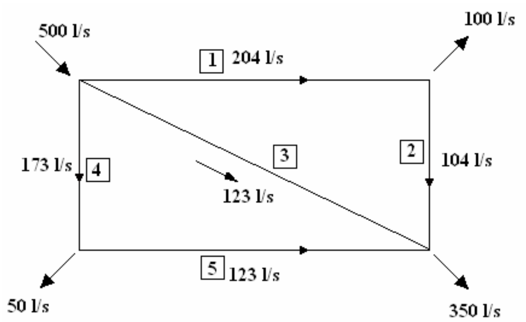
$$h_f = k_3\left\{\sqrt{\frac{h_f}{k_1}} + \sqrt{\frac{h_f}{k_2}}\right\}^2 = k_3\left\{\left(\frac{h_f}{k_1} + \frac{h_f}{k_2} + \frac{2h_f}{\sqrt{k_1k_2}}\right)\right\}$$

$$1 = k_3\left\{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{2}{\sqrt{k_1k_2}}\right)\right\}$$

$$k_3 = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{2}{\sqrt{k_1k_2}}}$$

(b) When the flow rates are expressed in litres per second and the head losses in metres, K values for the pipe systems shown are as given in the table. Under a particular set of inputs and demands the network experienced the flow rates indicated.

The head loss in the system was considered to be excessive and a second pipe was alongside pipe 3 so that they carried flow in parallel. The equivalent single pipe for these two pipes has k = 0.000818 $ms^2/litre^2$. When the pipe had been installed the pipe flows shown changed but the inputs and demands on the system remained the same.



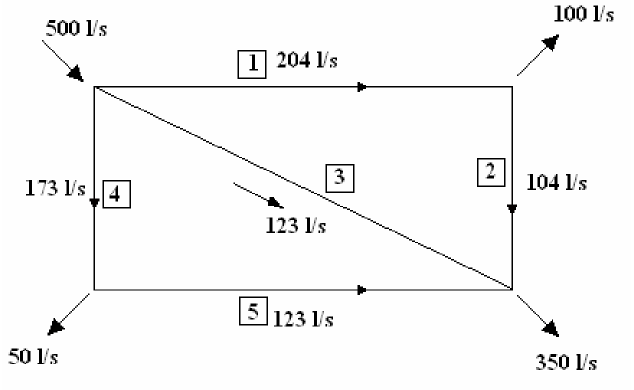
Use the flows shown as initially assumed flows and apply an iterative method of network analysis to determine the changed flows in the pipes. Make only two rounds of corrections to the initial flows.

Pipe	1	2	4	5
K ms^2/l^2	0.000570	0.012118	0.001698	0.006946

(Pipe 3 has K = 0.000818 in question)

SOLUTION PART B

The problem must be solved as two loops with a common pipe 3. Start with loop 1 with the flows shown. Data is shown for initial guess. Note clockwise flow is positive.

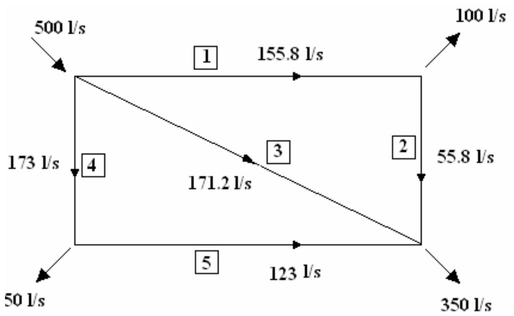


Starting data

First iteration loop 1 (pipes 1, 2 and 3)

PIPE	K	Q	h_f	h_f/Q
1	0.000570	204	23.7212	0.11628
2	0.012118	104	131.068	1.260
3	0.000818	-123	-12.376	0.1006
			142.4	1.4772

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{142.4}{2 \text{ x } 1.4772} = 48.2$$ Correct all flows in loop 1 by subtracting 48.2



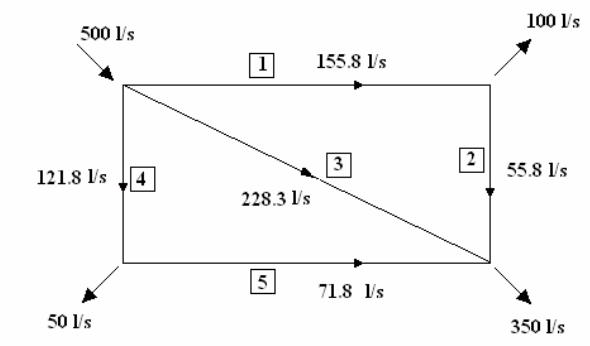
First correction shown above

First Iteration loop 2 (pipes 3,5 and 4)

PIPE	K	Q	h_f	h_f/Q
3	0.000570	171.2	23.97	0.1400
5	0.006946	-123	-105.08	-0.8544
4	0.001698	-173	-50.82	-0.2938
			-131.93	1.288

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{-131.93}{2 \text{ x } 1.288} = -51.208$$

Correct all flows in loop 2 by subtracting -51.2



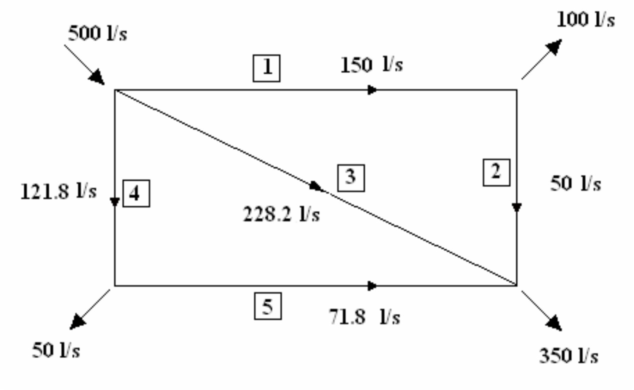
Second correction

Second iteration loop 1

PIPE	K	Q	h_f	h_f/Q
1	0.000570	155.8	13.8	0.0888
2	0.012118	55.8	37.7	0.676
3	0.003272	-222.4	-40.5	0.947
			11.1	0.947

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = \frac{11.1}{2 \text{ x } 0.947} = 5.9$$

Correct all flows in loop 1 by subtracting 5.9

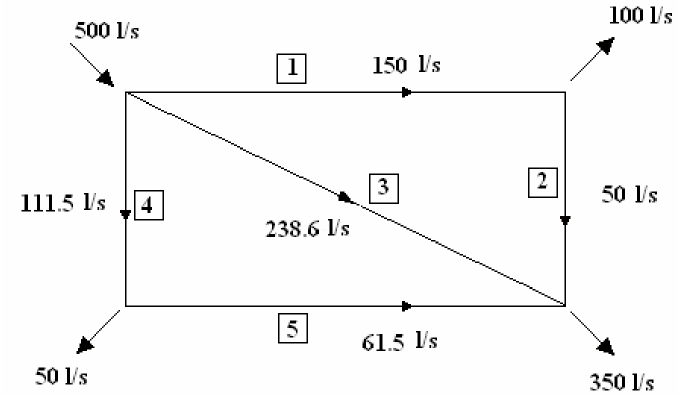


After third correction

Second iteration loop 2

PIPE	K	Q	h_f	h_f/Q
3	0.000570	228.27	42.6	0.187
5	0.006946	-71.8	-35.7	0.499
4	0.001698	-121.8	-25.2	0.207
			-18.4	0.892

$$\delta Q = \frac{\sum h_f}{2\sum h_f/Q} = -\frac{-18.4}{2 \text{ x } 0.892} = -10.3$$



Results after 2 iterations