SOLID MECHANICS
DYNAMICS
SCREW DRIVES

On completion of this short tutorial you should be able to do the following.

- Calculate the force needed to slide a load up or down an inclined plane.
- Describe a lead screw and Screw Jack.
- Calculate the effort and torque needed to raise and lower a load.
- Calculate the efficiency of a screw jack.

It is assumed that the student is already familiar with the following concepts.

- Coefficient of Friction.
- Resolution of forces.
- Lifting machines – force and velocity ratio

All these above may be found in the pre-requisite tutorials.

Contents

1. Introduction

The principle of a screw thread has been around for a long time. The friction between the mating threads is normally very large and it is unlikely that a nut would ever unscrew itself under the action of any axial force. To understand this we relate the problem to a block sliding on an inclined plane since a screw thread is in essence a spiral inclined plane. In this tutorial we are examining square screw threads such as used in lead screws and jacks.

2. Friction on Inclined Planes

You should already know that the coefficient of friction is defined as $\mu = F/R$ where F is the force parallel to the surface and R is the force normal to the surface.

Consider a block on an inclined plane at angle β to the horizontal. The weight acts vertically downwards. This must be resolved into two components parallel and perpendicular to the plane.

Resolving

$$R = W \cos \beta \quad \text{and } F_1 = W \sin \beta$$

If no other force is involved then the block will slide down the plane if F_1 is greater than the friction force.

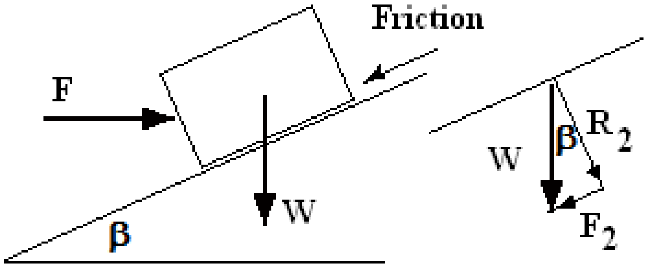

Figure 1

In this case $F_1 > \mu R$ or $F_1 > \mu W\cos \beta$

The block will just slide when $F_1 = \mu W\cos \beta$ so it follows that:

$$\mu = \frac{W\sin\beta}{W\cos\beta} = \tan\beta$$

In other formula we can substitute $\mu = \tan \beta$ even when the actual gradient is different.

WORKED EXAMPLE No. 1

A block rests on a plane and the angle is increased until it just slides. This angle is 13^o. Determine the coefficient of friction.

SOLUTION

$\mu = \text{Tan } \alpha = \text{Tan } 13^o = 0.231$

2.1 Sliding Up the Plane

Now consider the case of a block sliding under the action of a horizontal force such that the block slides up the plane. We must resolve the weight and the force parallel and perpendicular to the plane as shown.

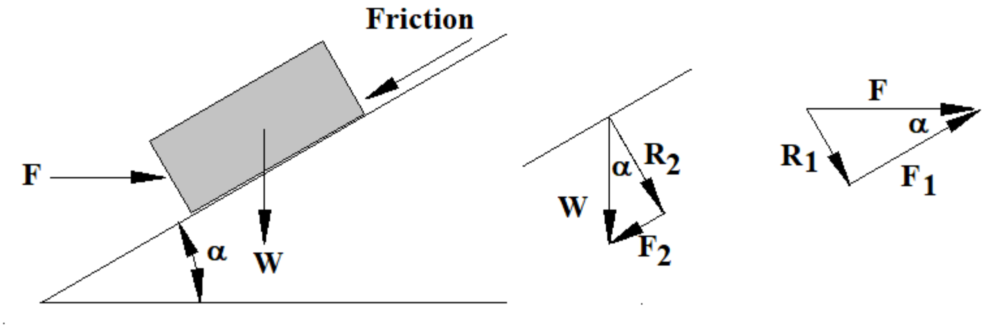

Figure 2

The total force acting parallel to the plane is $F_1 - F_2$ and the total reaction is $R = R_1 + R_2$

The block will just slide up the plane if $F_1 - F_2 = \mu (R_2 + R_1)$

$F_1 = F\cos\alpha$ $\qquad$ $F_2 = W\sin\alpha$ $\qquad$ $R_2 = W\cos\alpha$ $\qquad$ $R_1 = F\sin\alpha$

$$F\cos\alpha - W\sin\alpha = (W\mu\cos\alpha + F\mu\sin\alpha)$$

$$F(\cos\alpha - \mu\sin\alpha) = W(\mu\cos\alpha + \sin\alpha)$$

$$F = \frac{W(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)} = \frac{W(\mu + \tan\alpha)}{(1 - \mu\tan\alpha)}$$

Substitute $\mu = \tan\beta$

$$F = \frac{W(\tan\beta + \tan\alpha)}{(1 - \tan\beta\tan\alpha)}$$

A trig identity changes this to:

$$F = W\tan(\boldsymbol{\beta} + \boldsymbol{\alpha})$$

WORKED EXAMPLE No. 2

A block rests on a plane at 12^o to the horizontal. The weight is 80 N and the coefficient of friction is 0.4. Calculate the force that will just make it slide up the plane.

SOLUTION

$R_1 = F\sin 12^o$ $\qquad$ $R_2 = W\cos 12^o$ $\qquad$ $F_1 = F\cos 12^o$ $\qquad$ $F_2 = W\sin 12^o$

$R = (R_1 + R_2) = F\sin 12^o + W\cos 12^o$ $\qquad$ $F = F_1 - F_2 = F\cos 12^o - W\sin 12^o$

$$\mu = 0.4 = \frac{F}{R} = \frac{F\cos 12^o - W\sin 12^o}{F\sin 12^o + W\cos 12^o} = \frac{0.978F - 16.63}{0.2079F + 78.25}$$

$$47.93 = 0.895F \qquad F = 53.56 \text{ N}$$

The simple way is:

$$\beta = \tan^{-1}(0.4) = 21.8^o$$

$$F = W\tan(\beta + \alpha) = 80\tan(21.8 + 12) = 53.56 \text{ N}$$

2.2 Sliding Down the Plane

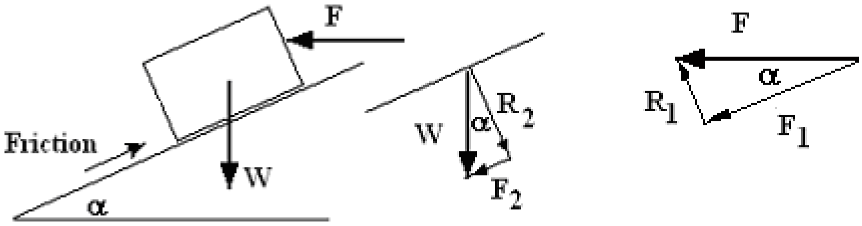

Figure 3

The force F acts to make the body slide down the plane.

The total force acting parallel to the plane is $F_1 + F_2$ and the total reaction is $R = R_2 - R_1$

The block will just slide down the plane if $F_1 + F_2 = \mu (R_2 - R_1)$

The same derivation as before shows that the force required to slide it down the plane is:

$$F = W \tan(\boldsymbol{\beta} - \boldsymbol{\alpha})$$

WORKED EXAMPLE No. 3

A block rests on a plane at 12^o to the horizontal. The weight is 80 N and the coefficient of friction is 0.4. Calculate the force that will just make it slide down the plane.

SOLUTION

$R_1 = F \sin 12^o$ $\quad R_2 = W \cos 12^o$

$F_1 = F \cos 12^o$ $\quad F_2 = W \sin 12^o$

$R = (R_2 - R_1) = W \cos 12^o - F \sin 12^o$

$F = F_1 + F_2 = \ F \cos 12^o + W \sin 12^o$

$$\mu = \frac{F}{R} = \frac{F \cos 12^o + W \sin 12^o}{W \cos 12^o - F \sin 12^o}$$

$$\mu = 0.4 = \frac{0.978\ F + 16.63}{78.25 - 0.2079\ F}$$

$$31.3 - 0.08316\ F = 0.978\ F + 16.63$$

$$14.67 = 1.061\ F$$

$$F = 13.82\ N$$

The simple way is:

$$\beta = \tan^{-1}(0.4) = 21.8^o$$

$$F = W \tan(\beta - \alpha) = 80 \tan(21.8 - 12) = 13.82\ N$$

3. *Application to Screw Thread*

The motion of two mating threads is the same as the previous problem. The vertical load is the thrust acting axially on the nut (e.g. the load on a screw jack). The angle of the plane is given by:

$$\tan\alpha = \frac{\text{pitch}}{\text{circumference}} = \frac{p}{\pi D}$$

3.1 Lead Screw

Lead Screw usually has a square thread. They are used to convert rotational motion into linear motion. The thread is rotated and the saddle moves in guides. This is used on many machines from lathes to linear electric actuators. The diagram shows a typical arrangement. The saddle carries a load and is moved up or down by rotation of the lead screw.

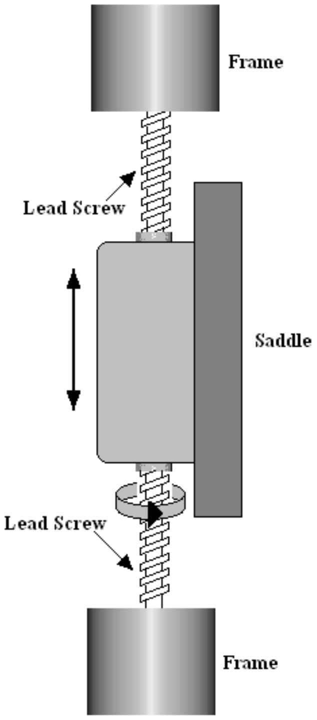

Figure 4

WORKED EXAMPLE No. 3

A lead screw has square threads with a pitch of 3 mm and a mean diameter of 12 mm. The coefficient of friction is 0.2. Calculate the torque needed to turn it when the load is 4 kN.

SOLUTION

The pitch is 3 mm and the circumference is 12π so the angle of the plane is

$$\alpha = \tan^{-1}\frac{p}{\pi D} = \tan^{-1}\frac{3}{12\pi} = 4.55^o$$

The friction angle is $\beta = \tan^{-1} 0.2 = 11.31^o$
The axial force is the force equivalent to the weight W.
The torque T is the product of the force F and radius at which it acts which is the radius of the thread (6 mm).

F = W tan (β + α) = 4 000 tan(15.86^o) = 1 136 N

T = F × radius = 1136 × 0.006 = 6.8 N m

3.2 Screw Jacks

Screw jacks work on the principle that the thread is rotated and screws or unscrews from the base to lift or lower the load.

One complete revolution of the screw raises the load by the pitch of the thread p.

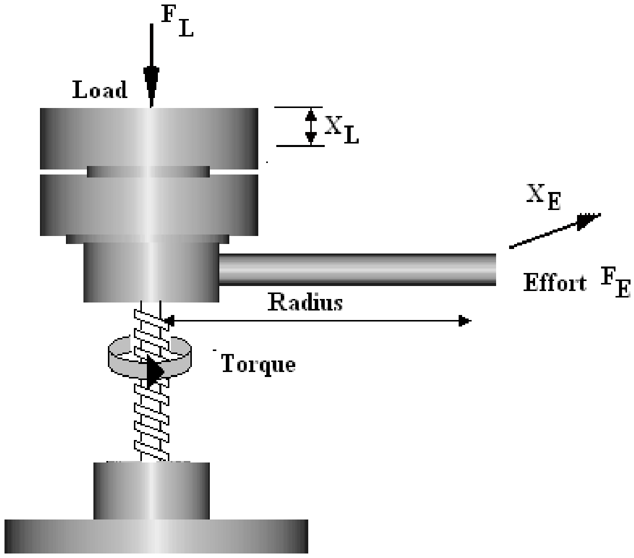

Figure 5

The distance moved by the effort $X_E = 2\pi R$ where R is the radius at which the effort is applied.

The distance moved by the load X_L= pitch = p

The Velocity Ratio (Force Ratio) is

$$\text{V.R.} = \frac{X_E}{X_L} = \frac{2\pi R}{p}$$

From the previous theory the force required at the thread is F **= F_L tan (β ± α)** depending whether the load is being raised or lowered. The Load is F_L

A torque of FD/2 is required to turn the screw where D is the thread diameter.

If a handle is used as shown with radius R then the effort is

$$F_E = \frac{F\,D}{2\,R}$$

The Force Ratio is

$$\text{F.R.} = \frac{F_L}{F_E} = \frac{2RF_L}{D\{F_L \tan(\beta \pm \alpha)\}} = \frac{2R}{D\tan(\beta \pm \alpha)}$$

The velocity ratio

$$\text{V.R.} = \frac{X_E}{X_L} = \frac{2\pi R}{p}$$

The efficiency is

$$\eta = \frac{\text{Work done by Load}}{\text{Work done by Effort}} = \frac{\text{F.R}}{\text{V.R}} = \frac{2R\,p}{D\tan(\beta \pm \alpha) \times 2\pi R} = \frac{p}{\pi D\tan(\beta \pm \alpha)}$$

When the load is being lifted

$$\eta = \frac{p}{\pi D \tan(\beta + \alpha)}$$

When being lowered

$$\eta = \frac{p}{\pi D \tan(\beta - \alpha)}$$

WORKED EXAMPLE No. 3

A screw jack has a thread with a pitch of 5 mm and diameter 25 mm. The coefficient of friction is 0.25. Calculate the efficiency when a load is raised. Go on to calculate the effort needed to raise 6 kN with a handle 500 mm long.

SOLUTION

$$\alpha = \tan^{-1}\frac{p}{\pi D} = \tan^{-1}\frac{5}{25\pi} = 3.64^{o}$$

$$\beta = \tan^{-1}\mu = \tan^{-1} 0.25 = 14^{o}$$

$$\eta = \frac{p}{\pi D \tan(\beta + \alpha)} = \frac{5}{\pi \times 25\tan(17.64^{o})} = 0.2 \text{ or } 20\%$$

$X_E = 2\pi \times 500 = 1\,000\pi$ mm $\qquad X_L = 5$ mm

$F_L = 6\,000$ N $\qquad$ Work done on load = $6\,000 \times 5$ mm = 30 000 N mm

$$\text{Work done by Effort} = F_E X_E = \frac{30\,000}{\eta} = \frac{30\,000}{0.2} = 150\,000 \text{ N mm}$$

$$F_E = \frac{150\,000}{2\pi R} = \frac{150\,000}{2\pi \times 500} = 47.7 \text{ N}$$

SELF ASSESSMENT EXERCISE No. 1

1. Calculate the horizontal force required to make a block weighing 60 N slide up a ramp inclined at 20^o to the ground given $\mu = 0.2$ (Answer 36.4 N)

2. The pitch of a thread on a screw jack is 6.38 mm and the mean diameter is 30 mm. Calculate the torque needed to raise a load 500 N. The coefficient of friction is 0. 51 (Answer 4.5 Nm)

3. A block weighing 600 N rests on an inclined plane at 12^o to the horizontal. The coefficient of friction is 0.41. Show that the formula to slide the box down the plane is **$F = W \tan(\beta - \alpha)$.** Calculate the horizontal force required to slide it DOWN the plane. (Answer 109 N)

4. A turnbuckle is used to adjust the tension in a wire to 600 N. Each end has a single start square thread with a mean diameter of 10 mm and pitch of 2 mm. The coefficient of friction is 0.2. Calculate the torque needed to tighten it.
   (Answer 1.6 Nm)

5. A screw Jack must raise a load of 500 kg. The thread has a pitch of 10 mm and a diameter of 50 mm. The coefficient of friction is 0.15. The effort is applied through a lever of radius 400 mm. Calculate the efficiency and the effort required to both raise the load and lower it.
   (29.5% and 66 N raising and 74.4% and 26.2 N lowering)

6. Calculate the coefficient of friction that just produces a zero effort in the last problem (i.e. the jack will screw down under the load with no effort) (0.064)

7. A lead screw and saddle arrangement is used to lift a load of 200 kg. The square thread has a pitch of 8 mm and a diameter of 30 mm. The coefficient of friction is 0.2. Calculate the torque needed to rotate the screw on raising and lowering.
   (8.53 and 3.33 Nm)

8. Calculate the coefficient of friction that just produces a zero torque in the last problem (i.e. the load lowers on its own) (0.085)