SOLID MECHANICS

DYNAMICS

COUPLED BODIES

On completion of this tutorial you should be able to do the following.

- Predict the common speed when two freely rotating bodies are suddenly coupled together.
- Calculate the energy lost that occurs when two freely rotating bodies are suddenly coupled together.

It is assumed that the student is already familiar with the following concepts.

Contents

1. *Introduction*

This short topic is about angular momentum and the way it is used to predict what happens when two rotating machines are coupled together. To do this you need to understand the law of conservation of momentum.

2. *Conservation of Momentum*

In the tutorial on linear momentum, the law of conservation of momentum was applied to bodies that collide or become joined. The law stated that "The total momentum before the event is equal to the total momentum after the event". In this tutorial we will apply the same law to rotating bodies that become joined (coupled).

Angular Moment is the product of angular velocity ω and moment of inertia I. The momentum of a flywheel is hence $I\omega$.

Consider two flywheels, initially rotating independently, that are connected by engaging a clutch.

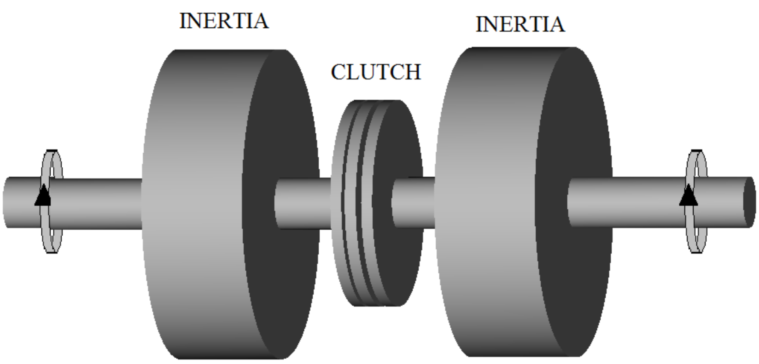

Figure 1

Total initial total momentum

$$I_1\omega_1 + I_2\omega_2$$

After connection they will have a single moment of inertia I_3 and a single speed ω_3 and the final momentum will be $I_3\omega_3$.

Because momentum is conserved we may equate the initial and final momentum.

$$I_1\omega_1 + I_2\omega_2 = I_3\omega_3.$$

And it follows that

$$I_3 = I_1 + I_2$$

Although momentum is conserved, energy is not and there will be a loss of energy during the coupling. This is due to friction in the clutch as the plates slip. The only energy involved here is angular kinetic energy.

$$\text{Angular Kinetic Energy} = \frac{I\omega^2}{2}$$

To appreciate this point, suppose two identical flywheels are rotating at the same speed but in opposite directions. When the clutch is engaged, both flywheels will be brought to a stop and the clutch will effectively be a brake that dissipates all the energy as friction.

WORKED EXAMPLE No. 1

A flywheel with a moment of inertia of 25 kg m^2 rotates at 20 rad/s. A second flywheel with a moment of inertia of 12 kg m^2 rotates at 30 rad/s. Calculate the speed when they are coupled by engaging a clutch. Also calculate the energy lost.

SOLUTION

$$I_3 = I_1 + I_2 = 25 + 12 = 37 \text{ kg m}^2$$

$$I_1\omega_1 + I_2\omega_2 = I_3\omega_3.$$

$$(25 \times 20) + (12 \times 30) = 37\omega_3$$

$$\omega_3 = 860/37 = 23.24 \text{ rad/s.}$$

Initial kinetic energy

$$KE_1 = \frac{I_1\omega_1^2}{2} + \frac{I_2\omega_2^2}{2}$$

$$KE_1 = \frac{25 \times 20^2}{2} + \frac{12 \times 30^2}{2} = 10\,400 \text{ J}$$

Final kinetic energy

$$KE_2 = \frac{I_3\omega_3^2}{2} = \frac{37 \times 23.24^2}{2} = 9\,994 \text{ J}$$

Energy lost = 10 400 – 9 994 = 406 Joules.

This would be lost in the clutch when the plates slip during the initial contact period.

SELF ASSESSMENT EXERCISE No.1

1. A flywheel with a moment of inertia of 10 kg m^2 rotates at 10 rad/s. A second flywheel with a moment of inertia of 12 kg m^2 is stationary. The two are coupled by engaging a clutch. Calculate the final common speed and the energy lost.
(4.545 rad/s and 272.7 J)

2. A flywheel with a moment of inertia of 5 kg m^2 rotates at 15 rad/s. A second flywheel with a moment of inertia of 3 kg m^2 rotates at 20 rad/s but in the opposite direction. Calculate the speed when they are coupled by engaging a clutch. Also calculate the energy lost.
(1.875 rad/s and 1148 J)