

# SOLID MECHANICS

## TUTORIAL – KINEMATICS - VELOCITY AND ACCELERATION DIAGRAMS

On completion of this tutorial you should be able to do the following.

- Describe a mechanism.
- Define relative and absolute velocity.
- Define relative and absolute acceleration.
- Define radial and tangential velocity.
- Define radial and tangential acceleration.
- Describe a four bar chain.
- Solve the velocity and acceleration of points within a mechanism.
- Use mathematical and graphical methods.
- Construct velocity and acceleration diagrams.
- Define the Coriolis Acceleration.
- Solve problems involving sliding links.

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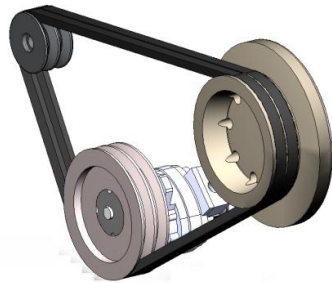
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## 1. Mechanism and Links

Mechanisms are used to transform motion from form to another. The transformation may produce a change in velocity, acceleration, force, torque, speed of rotation and so on. Here are some examples of mechanisms and a description of what they do.

### *Pulley Drives, Chain Drives and Gears*

These convert one speed and torque into another speed and torque.



PULLEY DRIVE



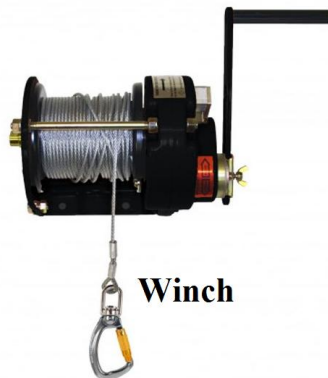
CHAIN DRIVE



GEAR DRIVE

### *Winch and a Rack and Pinion*

These convert angular motion into linear motion and linear motion into angular motion. They also convert force into torque or torque into force.

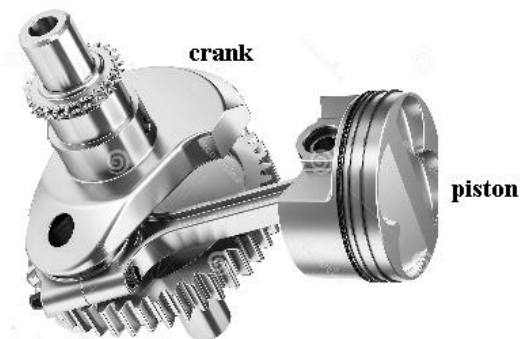
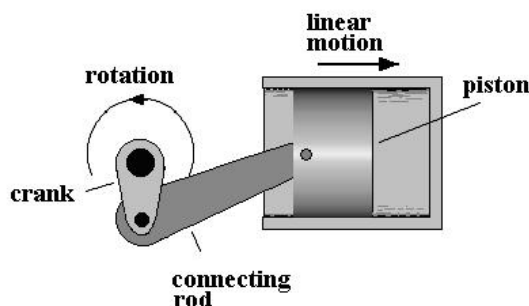


Winch



Rack and Pinion

### *Crank and Piston*

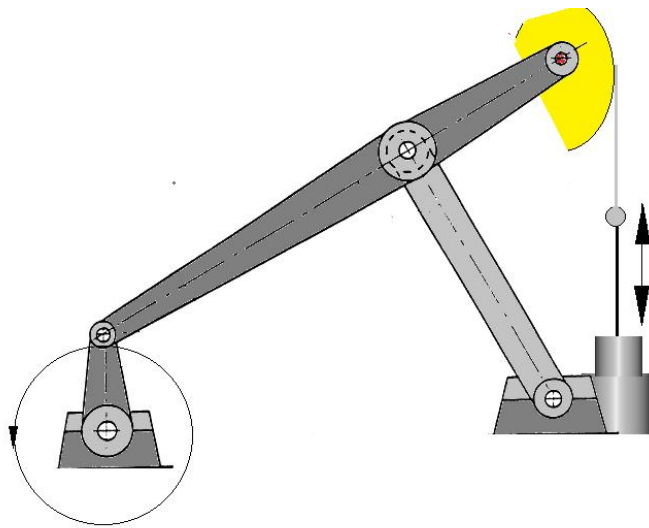


If the crank is turned, angular motion is converted into linear reciprocating motion of the piston and input torque is transformed into force on the piston. If the piston is forced to move, the linear motion is converted into rotary motion and the force into torque. The piston is a sliding joint and this is called *prismatic* in some fields of engineering such as robotics. The pin joints allow rotation of one part relative to another. These are also called *revolute* joints in other areas of engineering.



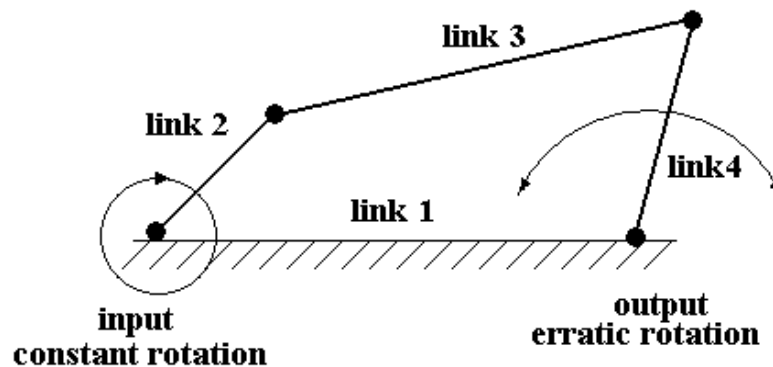
## Nodding Donkey

These are used for oil wells. The input rotates and the head nod up and down to operate the pump inside the well. Basically rotation is converted into a linear motion.



## 2. 4 Bar Mechanisms

You will find in these studies that the various parts of the mechanisms are referred to as links. It can be shown that all mechanisms are made up of a series of four links. The basic four bar link is shown below. When the input link rotates the output link may for example swing back and forth. With experience you should be able to identify the four bar chains in a mechanism. All the links shown are rigid links which means they may push or pull. It is possible to have links made of chain or rope which can only pull.



The **Grashof condition** for a four-bar linkage states that if the sum of the shortest and longest link is less than or equal to the sum of the remaining two links, then the shortest link can rotate fully with respect to a neighbouring link.

**The Grashof Classification** denotes the lengths of the 4 links as follows:

Input Crank - A      Output Crank - B      Fixed Link - C      **4<sup>th</sup> Link - D**

Define:       $L_1 = C + D - A - B$        $L_2 = B + C - A - D$        $L_3 = B + D - A - C$

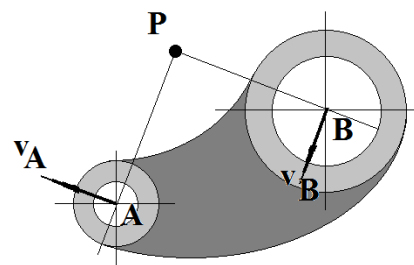
There are 4 classes of 4 bar mechanisms defined as:

$L_1$	$L_2$	$L_3$	<b>Input link</b>	<b>Output link</b>
Negative	Negative	Positive	Crank	Crank
Positive	Positive	Positive	Crank	Rocker
Positive	Negative	Negative	Rocker	Crank
Negative	Positive	Negative	Rocker	Rocker

Other classes have been produced based on defining the motion of a rocker.

### 2.1 Instantaneous Centre of Rotation

All links in a mechanism have an instantaneous centre of rotation. This is defined as the point (on or off the link) that has zero velocity at a particular instant of time. At this instant every part of the link is moving tangentially to any line drawn through the centre. For example the link shown has an instantaneous centre at point P. All other points (e.g. A and B) move normal to any line drawn through P at that instant. Point P probably moves on the plane



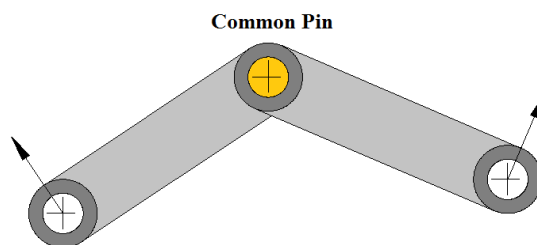
Here are two web sites that explain this more completely.

[http://engineering2.union.edu/~tchakoa/mer312/Lectures/Lecture06\\_IC.pdf](http://engineering2.union.edu/~tchakoa/mer312/Lectures/Lecture06_IC.pdf)

<https://www.geogebra.org/m/31786>

The instantaneous centre of rotation may be defined as:

- The point in one body about which some other body permanently or instantaneously rotates.
- A point where there is zero relative velocity between the two bodies such as a common pin joint



If there are n links the number of instant centres is:

$$P = \frac{n(n-1)}{2}$$

Hence for 4 bars ( $n = 4$ ) there are 6 centres.

## 2.2 Kennedy's Theorem

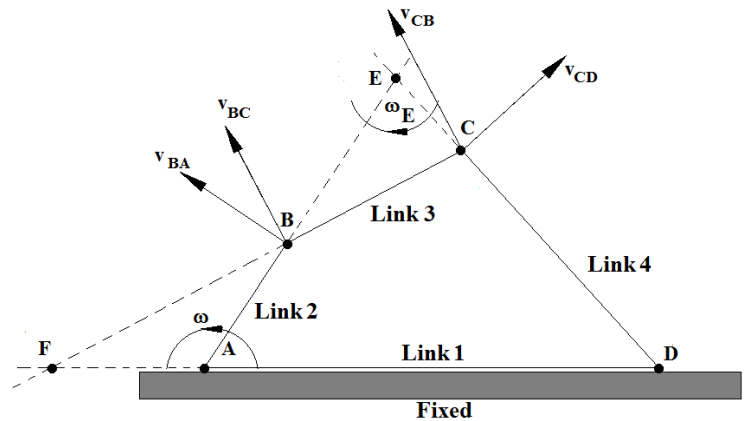
This states that the three instant centres shared by three rigid bodies in relative planar motion to another (whether or not connected) all lie on the same straight line.

The next figure shows a typical 4 bar chain. There are 4 links and they are designated by numbers and letters as follows:

**Link 1 - DA**   **Link 2 - AB**  
**Link 3 - BC**   **Link 4 - CD**

All the pin joints A, B, C and D are instant centres of rotation. There are two more E and F which are found by projecting the lines as shown. The tangential velocities are designated by the two letters for example  $v_{BA}$  means the velocity of B relative to A.

The key to the following is to realise that centre E is the common centre of rotation for links 3 and 4. Centre F is the common centre of rotation for links 2 and 4. In other publications a different notation is used to identify them such as  $P_{13}$  instead of E.



Let's relate the angular velocity  $\omega$  of link 2 to the length of the links. Every part of link 3 is rotating at angular velocity  $\Omega$  about point E. Tangential velocity is the product of radius and angular velocity so:

$$\omega_E = \frac{v_{BA}}{EB} \quad v_{BA} = \omega AB \quad \omega_E = \omega \times \frac{AB}{EB}$$

Similarly:

$$\omega_E = \frac{v_{CD}}{EC} = \omega \times \frac{AB}{EB} \quad v_{CD} = \frac{\omega (AB) (EC)}{(EB)}$$

From this you could calculate the angular velocity of link 4 about point D

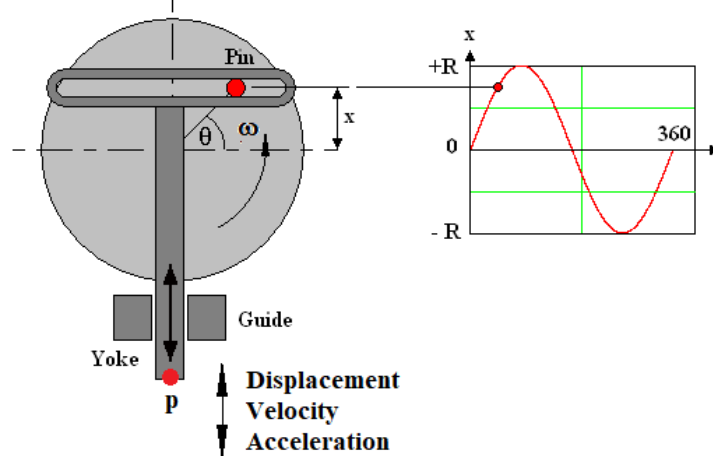
$$\omega_{ED} = \frac{v_{CD}}{DC}$$

The same idea can be applied to accelerations.

### 3. Displacement, Velocity and Acceleration Mathematical Analysis

In this section we will look at a purely mathematical method to solve the velocity and acceleration of points in a mechanism.

All parts of a mechanism have displacement, velocity and acceleration. A mechanism called the Scotch Yoke is examined next because it produces sinusoidal or harmonic motion. The wheel turns at a constant speed  $\omega$  rad/s and the yoke moves up and down.



The angle rotated is  $\theta = \omega t$ , linear displacement 'x', velocity 'v' and acceleration 'a' of point p is given as follows.

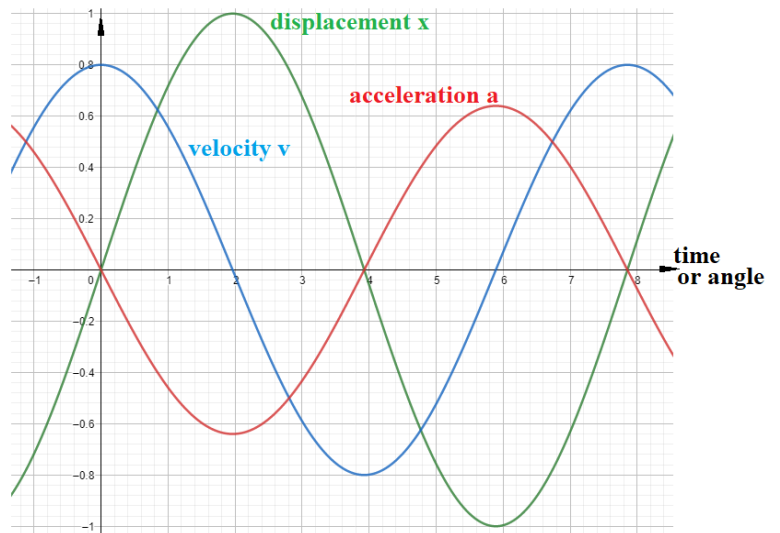
$$\text{Displacement} = x = R \sin(\omega t)$$

$$\text{Velocity} = v = \frac{dx}{dt} = \omega R \cos(\omega t)$$

$$\text{Acceleration} = a = \frac{dv}{dt}$$

$$a = -\omega^2 R \sin(\omega t)$$

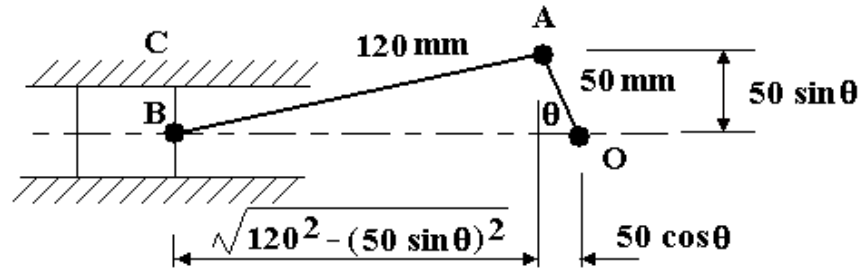
The graphical plot of these are typically as shown.



The values can be calculated for any angle or moment of time. The acceleration could then be used to calculate the inertia force needed to accelerate and decelerate the link. Clearly it is the maximum values that are needed. Other mechanisms can be analysed mathematically in the same way but it is more difficult. The starting point is to derive the equation for displacement with respect to angle or time and then differentiate twice to get the acceleration. Without the aid of a computer to do this, the mathematics is normally much too difficult and a graphical method should be used as described later.

### WORKED EXAMPLE No. 1

A crank, con rod and piston mechanism is shown below. Determine the maximum acceleration of the piston when the crank speed is 30 rev/min clockwise.



### SOLUTION

When  $\theta = 0$  the piston will be furthest left at a distance of 170 mm from point O. Take this as the reference point and measure displacement  $x$  from there.

Remember that  $\theta = \omega t$  and  $\omega = 2\pi \times 30/60 = 3.142$  rad/s. The displacement is then:

$$x = 170 - \{50\cos(\omega t)\} - \left[ \sqrt{120^2 - \{50\cos(\omega t)\}^2} \right]$$

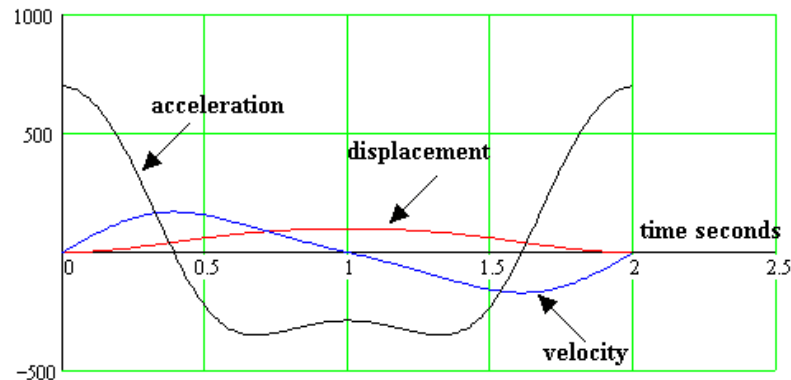
Differentiate to get the velocity

$$v = 50\omega \sin(\omega t) + \frac{2500 \omega \sin(\omega t)\cos(\omega t)}{\sqrt{14400 - 2500 \sin^2(\omega t)}}$$

Differentiate again to get the acceleration.

$$a = 50 \omega^2 \cos(\omega t) + \frac{6250000 \omega^2 \sin^2(\omega t)\cos(\omega t)^2}{\{14400 - 2500 \sin^2(\omega t)\}^{\frac{3}{2}}} + \frac{2500 \omega^2 \cos(\omega t)^2}{\sqrt{14400 - 2500 \sin^2(\omega t)^2}} - \frac{2500 \omega^2 \sin(\omega t)^2}{\sqrt{14400 - 2500 \sin^2(\omega t)^2}}$$

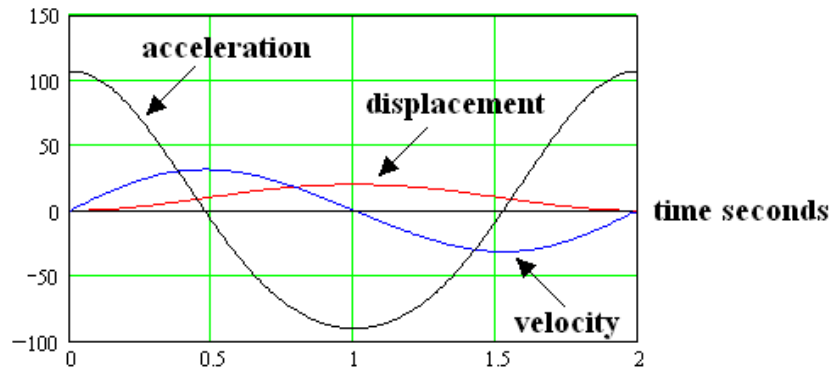
The diagram shows a plot of displacement, velocity and acceleration against angle. It should be noted that none of them are sinusoidal and not harmonic (in particular, the acceleration).





The units are all in mm and seconds. The above was done with a computer package. Plotting the above functions over a complete rotation shows that the maximum acceleration occurs at  $t = 0$  ( $\theta = 0$ ) and evaluating gives an answer of  $700 \text{ mm/s}^2$ .

If the radius of the crank is small in comparison to the length of the connecting rod, the motion becomes close to sinusoidal. To illustrate this, here is the plot with the crank radius reduced to 10 mm. The acceleration is now almost a cosine curve.



Solving these problems mathematically is difficult so we will now look at a graphical method. First we need some definitions.

#### 4. Velocity Diagrams

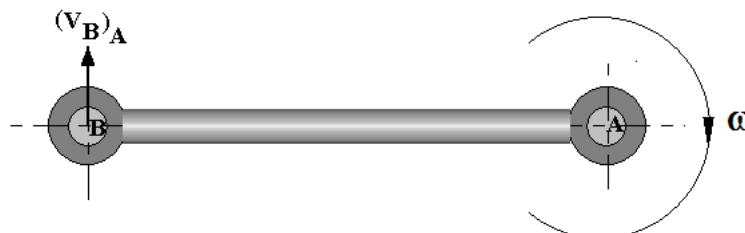
This section involves the construction of diagrams which needs to be done accurately and to a suitable scale. Students should use a drawing board, ruler, compass, protractor and triangles and possess the necessary drawing skills. If you have a computer with suitable drawing software you may use that. The problems can also be solved with trigonometry but you need to sketch the diagrams in order to do this.

##### 4.1 Absolute and Relative Velocity

An absolute velocity is the velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving). Relative velocity is the velocity of a point measured relative to another that may itself be moving.

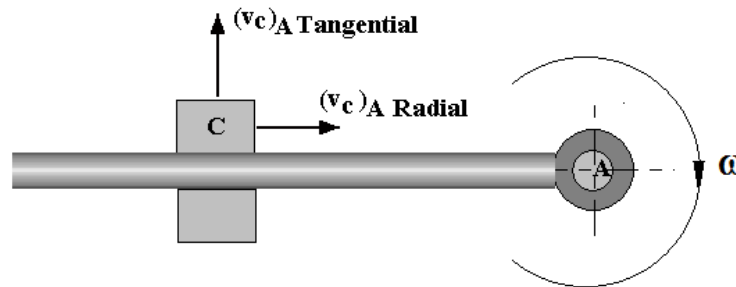
##### 4.2 Tangential Velocity

Consider a link A B pinned at A and revolving about A at angular velocity  $\omega$ . Point B moves in a circle relative to point A but its velocity is always tangential and hence at  $90^\circ$  to the link. A convenient method of denoting this tangential velocity is  $(v_B)_A$  meaning the velocity of B relative to A. This method is not always suitable.



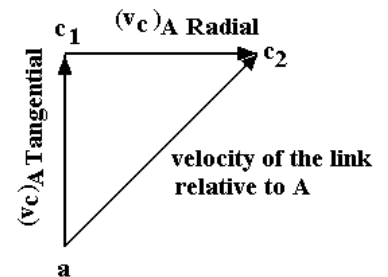
### 4.3 Radial Velocity

Consider a sliding link C that can slide on link AB. The direction can only be radial relative to point A as shown. If the link AB rotates about A at the same time then link C will have radial and tangential velocities.



Note that both the tangential and radial velocities are denoted the same so the tags radial and tangential are added.

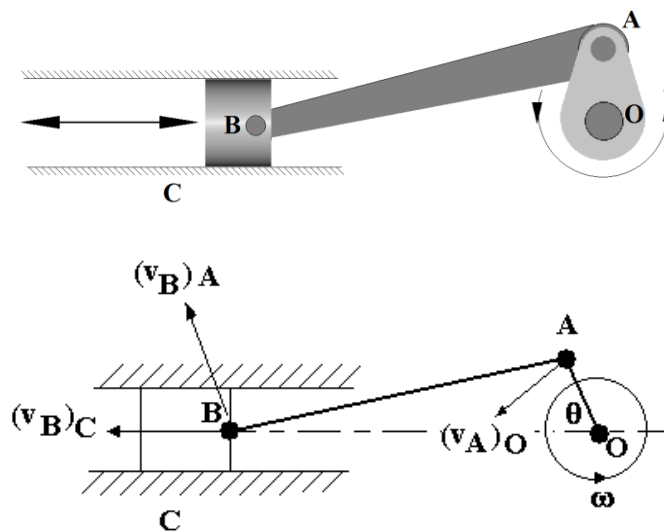
The sliding link has two relative velocities, the radial and the tangential. They are normal to each other and the true velocity relative to A is the vector sum of both added as shown. *Note that lower case letters are used on the vector diagrams.* The two vectors are denoted by  $c_1$  and  $c_2$ . The velocity of link C relative to point A is the vector  $a_{c_2}$ .



## 5. Crank, Connecting Rod and Piston (Slider Crank Chain)

### 5.1 Velocity Diagram

Consider this mechanism again. Let's freeze the motion (snap shot) at the position shown. The diagram is called a space diagram.



Every point on every link has a velocity through space. First we label the centre of rotation, (usually the letter O). Point A can only move in a tangential direction so the velocity of A relative to O is also its absolute velocity and the vector is normal to the crank and it is designated  $(v_A)_O$ . (Note the rotation is anticlockwise).

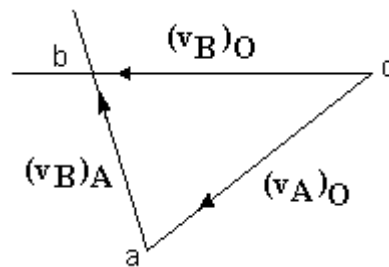
Now suppose that you are sat at point A and everything else moves relative to you. Looking towards B, it would appear the B is rotating relative to you (in reality it is you that is rotating) so it has a tangential velocity denoted  $(v_B)_A$ . The direction is not always obvious except that it is normal to the link.

Consider the fixed link OC. Since both points are fixed there is no velocity between them so:

$$(v_C)_O = 0$$

Next consider that you at point C looking at point B. Point B is a sliding link and will move in a straight line in the direction fixed by the slider guides and this is velocity  $(v_B)_C$ . It follows that the velocity of B seen from O is the same as that seen from C so  $(v_B)_C = (v_B)_O$

The absolute velocity of B is  $(v_B)_C = (v_B)_O$  and this must be the vector sum of  $(v_A)_O$  and  $(v_B)_A$  and the three vectors must form a closed triangle as shown. The velocity of the piston must be in the direction in which it slides (conveniently horizontal here). This is a velocity diagram.



## 5.2 Methodology

First calculate the tangential velocity  $(v_A)_O$  from  $v = \omega \times \text{radius} = \omega \times OA$

Draw the vector o - a in the correct direction (note lower case letters).

We know that the velocity of B relative to A is to be added so the next vector ab starts at point a. At point a draw a line in the direction normal to the connecting rod but of unknown length.

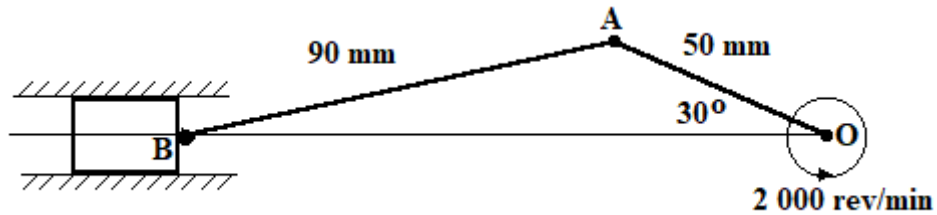
We know that the velocity of B relative and absolute to O is horizontal so the vector ob must start at a. Draw a horizontal line (in this case) through o to intersect with the other line. This is point b. The vectors ab and ob may be measured or calculated. Usually it is the velocity of the slider that is required.

In a design problem, this velocity would be evaluated for many different positions of the crank shaft and the velocity of the piston determined for each position.

Remember that the slider direction is not always horizontal and the direction of o - b must be the direction of sliding.

## WORKED EXAMPLE No. 2

The mechanism shown has a crank 50 mm radius which rotates at 2 000 rev/min. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.



## SOLUTION

Note the diagrams are not drawn to scale. The student should do this using a suitable scale for example 1 cm = 1 m/s. This is important so that the direction at 90° to the link AB can be transferred to the velocity diagram.

Angular speed of the crank

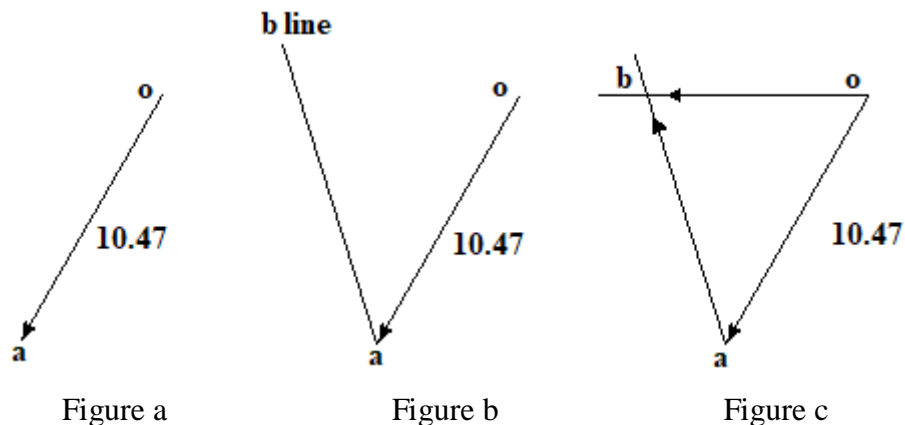
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2\,000}{60} = 209.4 \text{ rad/s}$$

$$(v_A)_O = \omega \times \text{radius} = 209.4 \times 0.05 = 10.47 \text{ m/s.}$$

First draw vector oa (diagram a).

Next add a line in the direction ab (diagram b)

Finally add the line in the direction of ob to find point b and measure ob to get the velocity (diagram C).



The velocity of B relative to O is 7 m/s.

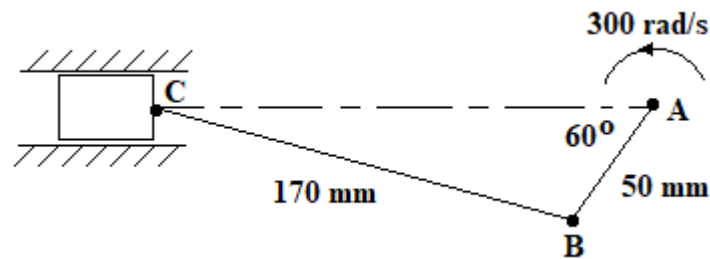
The tangential velocity of B relative to A is the vector ab and this gives 9.2 m/s.

The angular velocity of B about A is found by dividing by the radius (length of AB).

$\omega$  for AB is then  $9.2/0.09 = 102.2 \text{ rad/s}$ . (note this is relative to A and not an absolute angular velocity)

### WORKED EXAMPLE No. 3

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the velocity of the piston. The diagram is not drawn to scale.

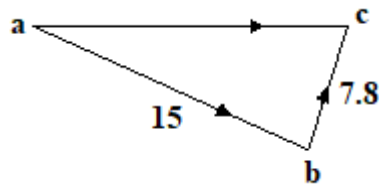


### SOLUTION

First calculate the tangential velocity of B relative to A.

$$(v_B)_A = \omega \times \text{radius} = 300 \times 0.05 = 15 \text{ m/s.}$$

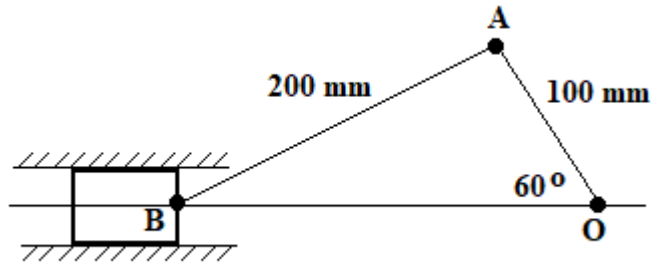
Next draw the velocity diagram and determine the velocity of C relative to B.



From the velocity diagram  $(v_C)_B = 7.8 \text{ m/s}$

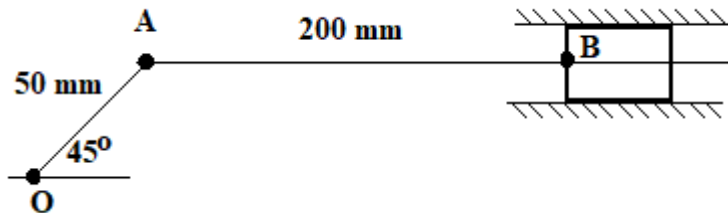
**SELF ASSESSMENT EXERCISE No. 1**

1. Find the velocity of the piston and the angular velocity of AB about point A. The crank OA rotates anti-clockwise at 3 000 rev/min.



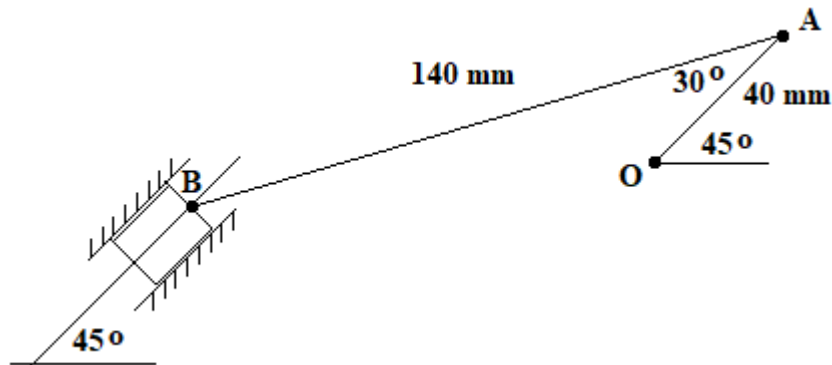
Answer 34 m/s and 21.25 rad/s

2. The crank revolves clockwise at 300 rev/min. Note that the vector ob is still horizontal because the piston can only slide horizontally relative to O. Also the rotation of the crank is opposite to the previous cases so the direction of oa is down to the right.



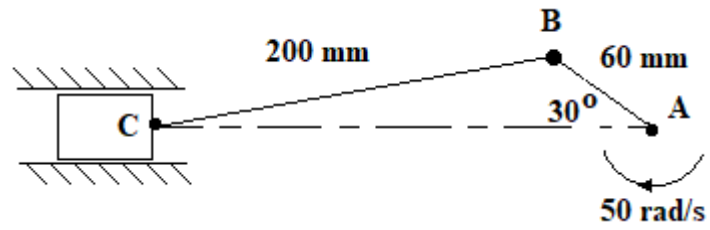
Answer 1.11 m/s to the right and 5.55 rad/s

3. The crank O-A rotates at 200 rev/min clockwise. Note the vector ob is at 45° to the horizontal as the piston must slide in this direction.



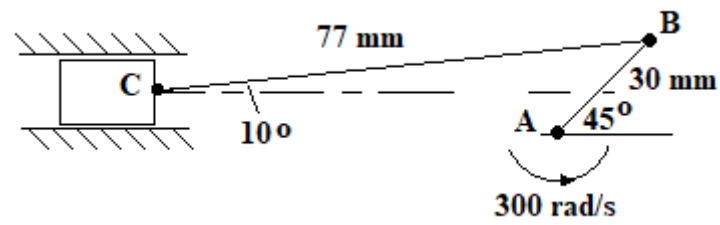
Answer 0.49 m/s and 6.92 rad/s.

4. Solve the velocity of the piston. You should draw the space diagram out accurately first.



(Ans. 1.89 m/s)

5. Solve the velocity of the piston. You should draw the space diagram out accurately first.



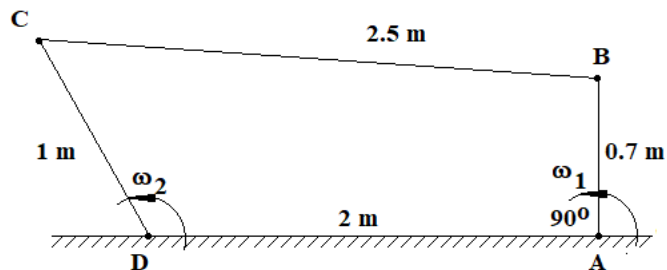
(Ans. 5.259 m/s)

## 6. Other 4 Bar Chain Velocity Diagrams

The basic 4 bar chain is shown and the description was given earlier. Let's look at solving the velocity diagrams next.

### WORKED EXAMPLE No. 4

The input link of the chain shown rotates at a constant speed of 500 rev/min. Find the angular velocity of the output link at the instant shown.



### SOLUTION

First calculate  $\omega_1$

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 500}{60} = 52.36 \text{ rad/s}$$

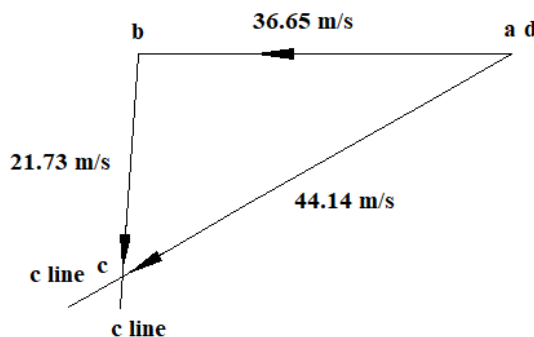
Next calculate the velocity of point B relative to A.

$$(V_B)_A = \omega_1 \times AB = 52.36 \times 0.7 = 36.65 \text{ m/s.}$$

Draw a vector a to b 36.65 long to an appropriate scale horizontally at right angles to link AB.

Next draw the direction of velocity C relative to B at right angles to the link BC passing through point b on the velocity diagram.

Finally draw a line at right angles to link CD passing through point a (which is the same as point d). Point c is where the two lines intersect,



Determine velocity cd by measurement or any other method. The velocity of point C relative to D and is 44.35 m/s.

Convert this into angular velocity by dividing the length of the link DC into it.

$$\omega_2 = 44.35/0.7 = 63.36 \text{ rad/s (you can deduce that it is anticlockwise)}$$

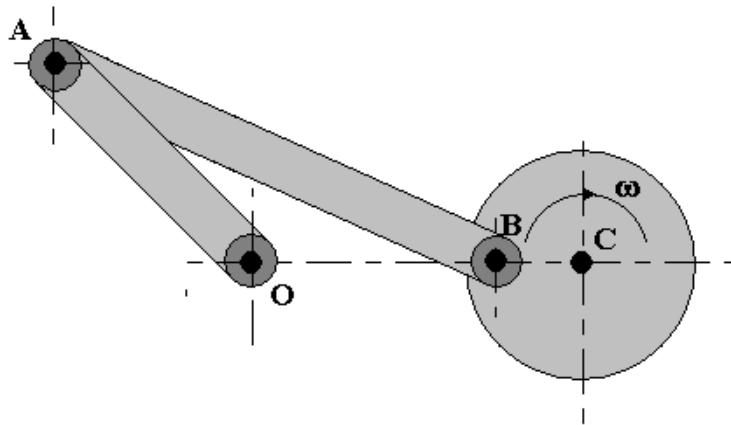


### WORKED EXAMPLE No. 5

The diagram shows a “rocking lever” mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centres. The wheel rotates clockwise at a uniform angular velocity ( $\omega$ ) of 100 rad/s. For the configuration shown, determine the angular velocity of the link AB and the absolute velocity of point A.

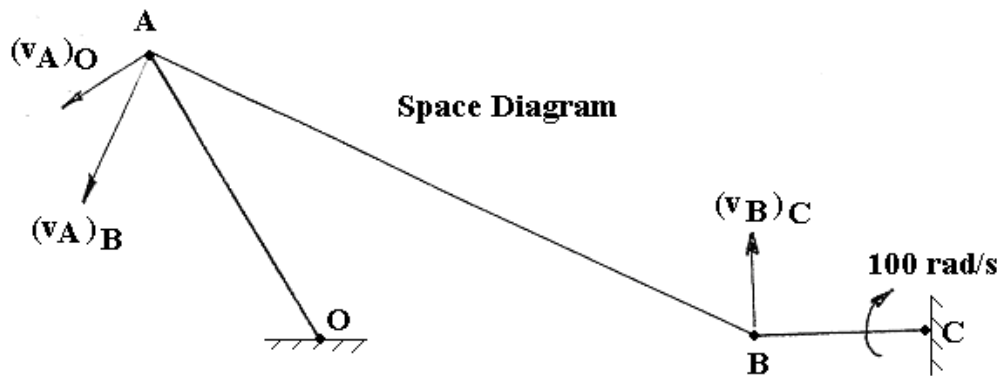
The lengths of the links are as follows.

$$BC = 25 \text{ mm} \quad AB = 100 \text{ mm} \quad OA = 50 \text{ mm} \quad OC = 90 \text{ mm}$$



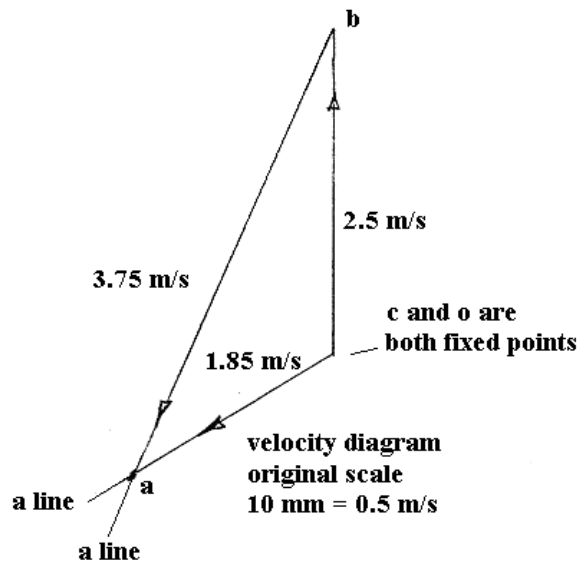
### SOLUTION

The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.



Next calculate the velocity of point B relative to C and construct the velocity diagram.

$$(v_B)_C = \omega \times \text{radius} = 100 \times 0.025 = 2.5 \text{ m/s}$$



Scale the following velocities from the diagram.

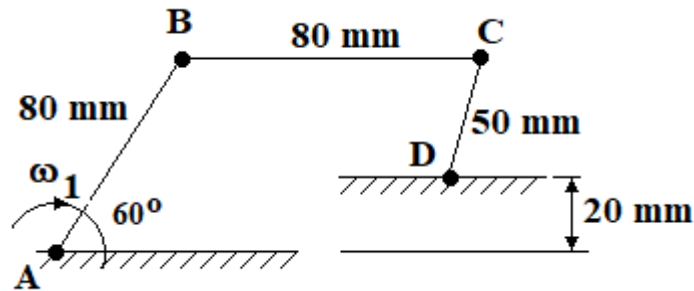
$$(v_A)_O = 1.85 \text{ m/s} \text{ \{ answer (i) \}} \quad (v_A)_B = 3.75 \text{ m/s}$$

Angular velocity = tangential velocity/radius

$$\text{For link AB, } \omega = 3.75/0.1 = 37.5 \text{ rad/s.}$$

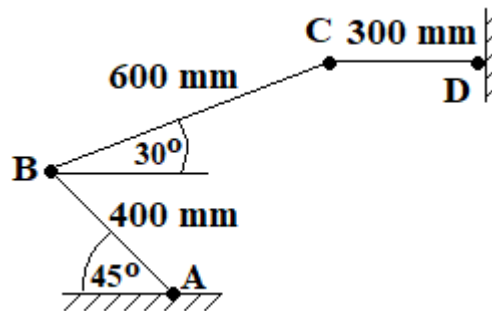
**SELF ASSESSMENT EXERCISE No. 2**

1. Determine the angular velocity of the link DC. The input rotates at 500 rev/min. Link BC is horizontal.



(Ans. 76 rad/s clockwise.)

2. The input link AB rotates at 60 rev/min in a clockwise direction.



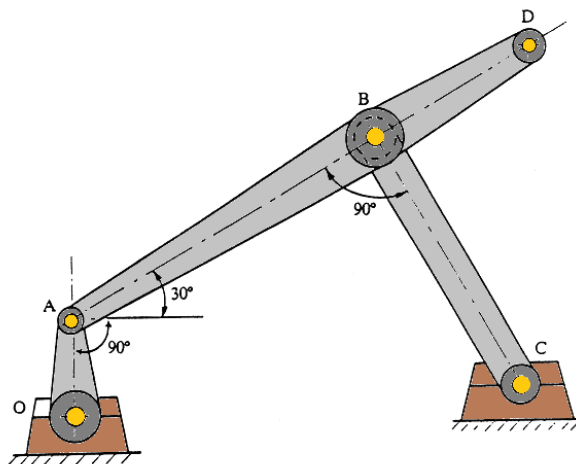
(Ans. 16 rad/s)

3. The diagram shows the instantaneous position of a mechanism in which member OA rotates in an anticlockwise direction with an angular velocity of 100 rad/s. BD is a continuation of the rigid link AB. The links have the following lengths.

OA – 30 mm    BC – 90 mm    AD – 168 mm    AB – 1120 mm

Determine the following.

- i. The velocities of points A, B and D (1.5 m/s, 2.6 m/s and 2.7 m/s)



## 7. Acceleration Diagrams

It is important to determine the acceleration of links because acceleration produces inertia forces in the link which stress the component parts of the mechanism. Accelerations may be relative or absolute in the same way as described for velocity. Acceleration diagrams are constructed in a similar way to velocity diagrams and this will be demonstrated in the examples that follow.

Sliding links have linear acceleration but we need to consider two other forms of acceleration, tangential and radial. Centripetal acceleration is an example of radial. In more advanced studies you will come across Coriolis acceleration which applies to links that slide and rotate.

### 7.1 Centripetal Acceleration

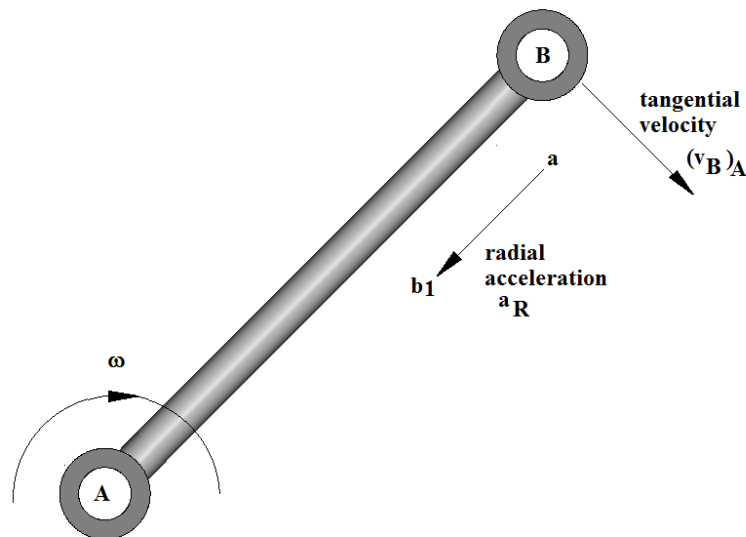
A point rotating about a centre at radius  $R$  has a tangential velocity  $v$  and angular velocity  $\omega$  and it is continually accelerating towards the centre even though it never moves any closer. This is centripetal acceleration and it is caused by the constant change in direction. It follows that the end of any rotating link will have a centripetal acceleration towards the opposite end.

The relevant equations are:  $v = \omega R$        $a = \omega^2 R$  or  $a = v^2/R$ .

The construction of the vector for radial acceleration causes confusion so the rules must be strictly followed. Consider the link  $AB$ . The velocity of  $B$  relative to  $A$  is tangential  $(v_B)_A$ .

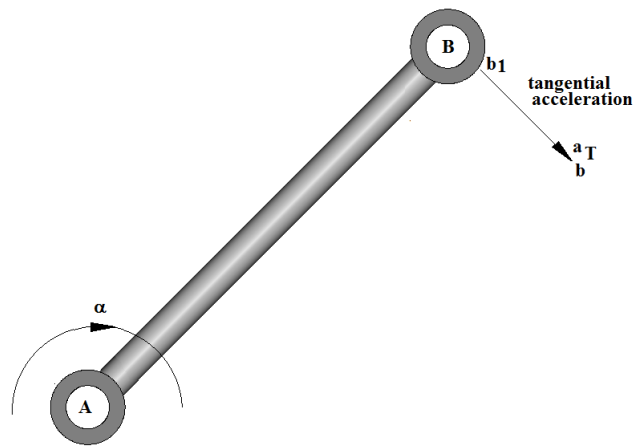
The centripetal acceleration of  $B$  relative to  $A$  is in a radial direction so a suitable notation might be  $a_R$ . It is calculated using  $a_R = \omega^2 \times AB$  or  $a_R = v^2/AB$ .

**Note the direction is towards the centre of rotation but the vector starts at  $a$  and ends at  $b_1$ .** It is very important to get this the right way round otherwise the complete diagram will be wrong.



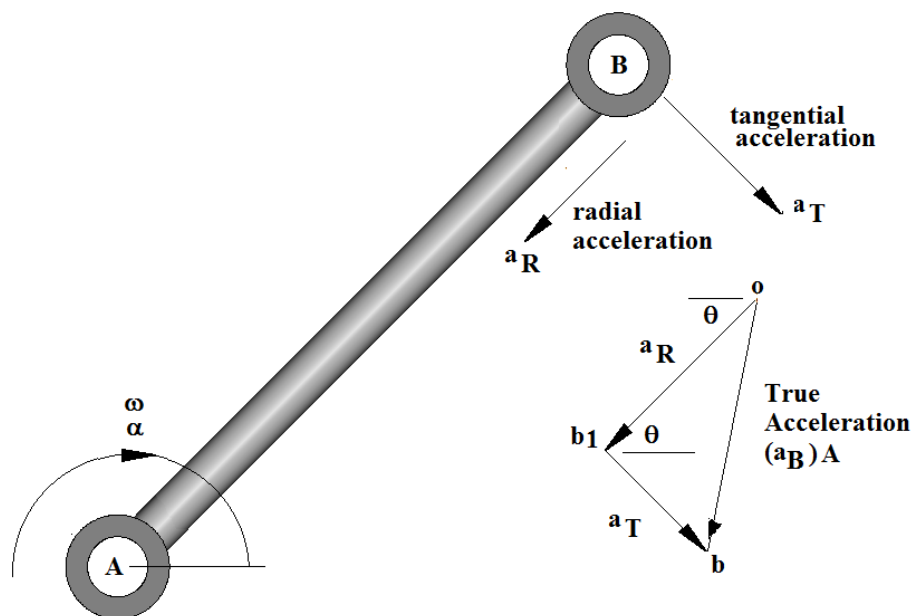
## 7.2 Tangential Acceleration

Tangential acceleration only occurs if the link has an angular acceleration  $\alpha \text{ rad/s}^2$ . Consider a link AB with an angular acceleration about A.



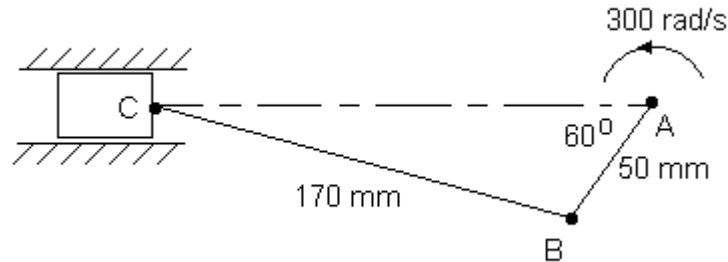
Point B will have both radial and tangential acceleration relative to point A. The true acceleration of point B relative to A is the vector sum of them. This will require an extra point. We will use  $b_1$  and  $b$  on the vector diagram as shown.

Point B is accelerating around a circular path and its direction is tangential (at right angles to the link). It is designated  $a_T$  and calculated using  $a_T = \alpha \times AB$ . The vector starts at  $b_1$  and ends at  $b$ . The choice of letters and notation are arbitrary but must be logical to aid and relate to the construction of the diagram.



### WORKED EXAMPLE No. 6

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the acceleration of the piston and the angular acceleration of the link BC. The diagram is not drawn to scale.

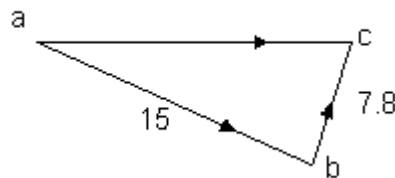


### SOLUTION

First calculate the tangential velocity of B relative to A.

$$(v_B)_A = \omega \times \text{radius} = 300 \times 0.05 = 15 \text{ m/s.}$$

Next draw the velocity diagram and determine the velocity of C relative to B.



From the velocity diagram  $(v_C)_B = 7.8 \text{ m/s}$

Next calculate all accelerations possible and construct the acceleration diagram to find the acceleration of the piston.

The tangential acceleration of B relative to A is zero in this case since the link has no angular acceleration ( $\alpha = 0$ ).

The centripetal acceleration of B relative to A

$$a_R = \omega^2 \times AB = 300^2 \times 0.05 = 4500 \text{ m/s}^2.$$

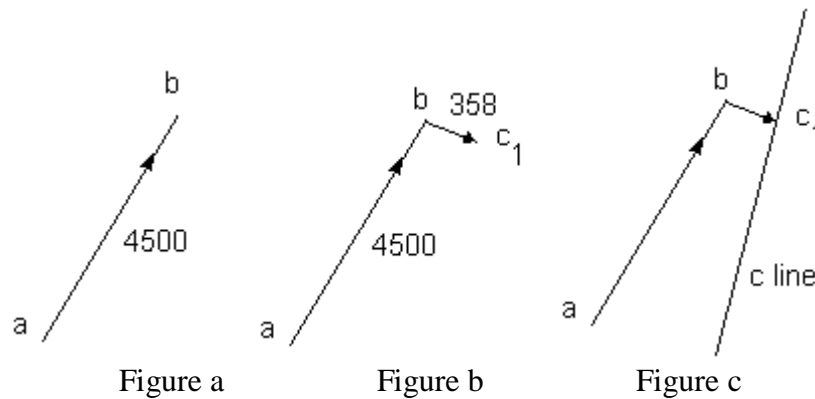
The tangential acceleration of C relative to B is unknown.

The centripetal acceleration of C to B

$$a_R = v^2/BC = 7.8^2 / 0.17 = 357.9 \text{ m/s}^2.$$

The stage by stage construction of the acceleration diagram is as follows.

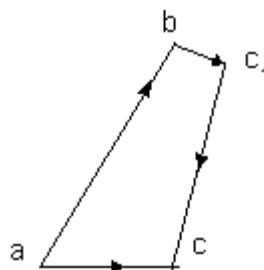
First draw the centripetal acceleration of link AB (Fig. a). There is no tangential acceleration so designate it  $ab$ . Note the direction is the same as the direction of the link towards the centre of rotation but it starts at a and ends at b.



Next add the centripetal acceleration of link BC (Figure b). Since there are two accelerations for point C designate the point  $c_1$ . Note the direction is the same as the direction of the link towards the centre of rotation.

Next add the tangential acceleration of point C relative to B (Figure c). Designate it  $c_1 c$ . Note the direction is at right angles to the previous vector and the length is unknown. Call the line a c line.

Next draw the acceleration of the piston (figure d) which is constrained to be in the horizontal direction. This vector starts at a and must intersect the c line. Designate this point c.



The acceleration of the piston is vector  $ac$  so  $(a_C)_B = 1505 \text{ m/s}^2$ .

The tangential acceleration of C relative to B is  $c_1 c = 4000 \text{ m/s}^2$ .

At the position shown the connecting rod has an angular velocity and acceleration about its end even though the crank moves at constant speed.

The angular acceleration of BC is the tangential acceleration divided by the length BC.

$$\alpha_{(BC)} = 4000 / 0.17 = 23529 \text{ rad/s}^2.$$

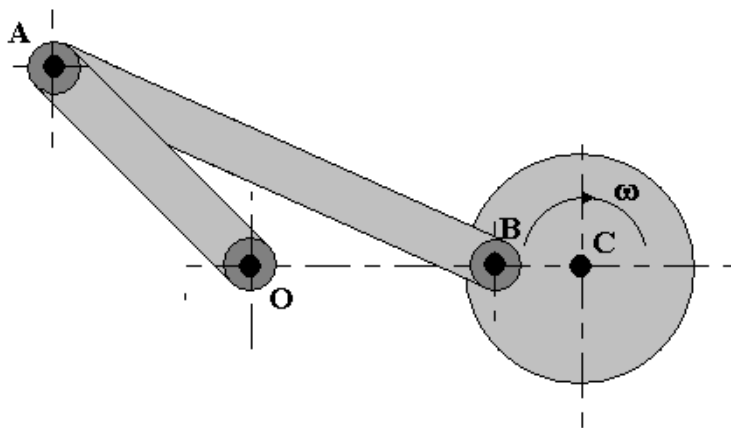
### WORKED EXAMPLE No. 7

The diagram shows a “rocking lever” mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centres. The wheel rotates clockwise at a uniform angular velocity ( $\omega$ ) of 100 rad/s. For the configuration shown, determine the following.

- (i) The angular velocity of the link AB and the absolute velocity of point A.
- (ii) The centrifugal accelerations of BC, AB and OA.
- (iii) The magnitude and direction of the acceleration of point A.

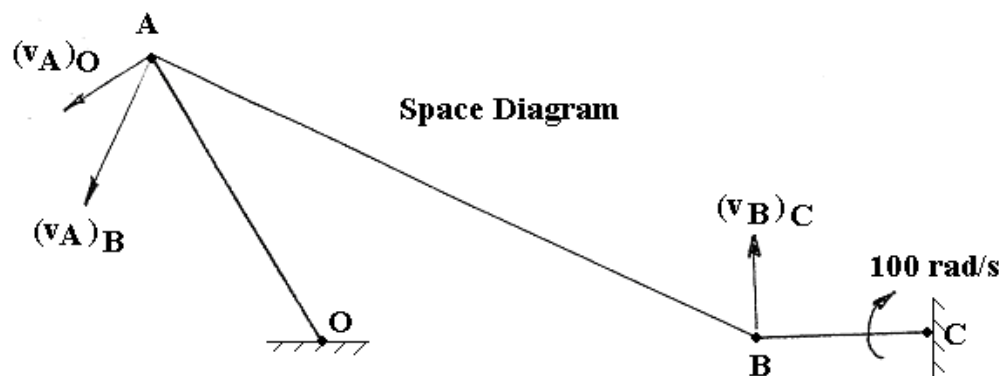
The lengths of the links are as follows.

$$BC = 25 \text{ mm} \quad AB = 100 \text{ mm} \quad OA = 50 \text{ mm} \quad OC = 90 \text{ mm}$$



### SOLUTION

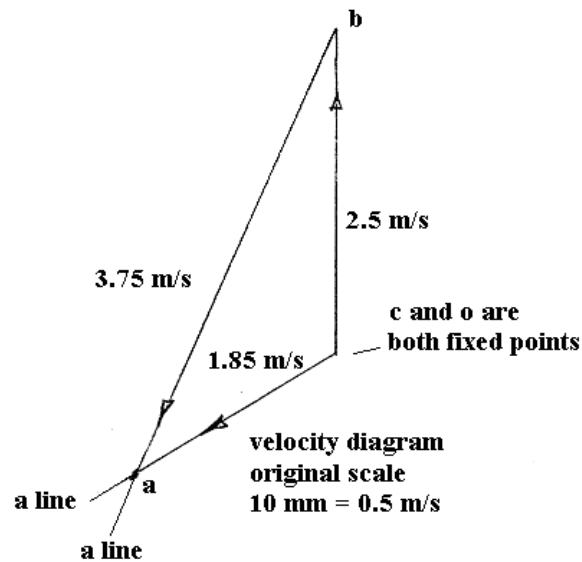
The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.



Next calculate the velocity of point B relative to C and construct the velocity diagram.



$$(v_B)_C = \omega \times \text{radius} = 100 \times 0.025 = 2.5 \text{ m/s}$$



Scale the following velocities from the diagram.

$$(v_A)_O = 1.85 \text{ m/s} \text{ \{answer (i)\}} \quad (v_A)_B = 3.75 \text{ m/s}$$

Angular velocity = tangential velocity/radius

$$\text{For link AB, } \omega = 3.75/0.1 = 37.5 \text{ rad/s. \{answer (i)\}}$$

Next calculate all the accelerations possible.

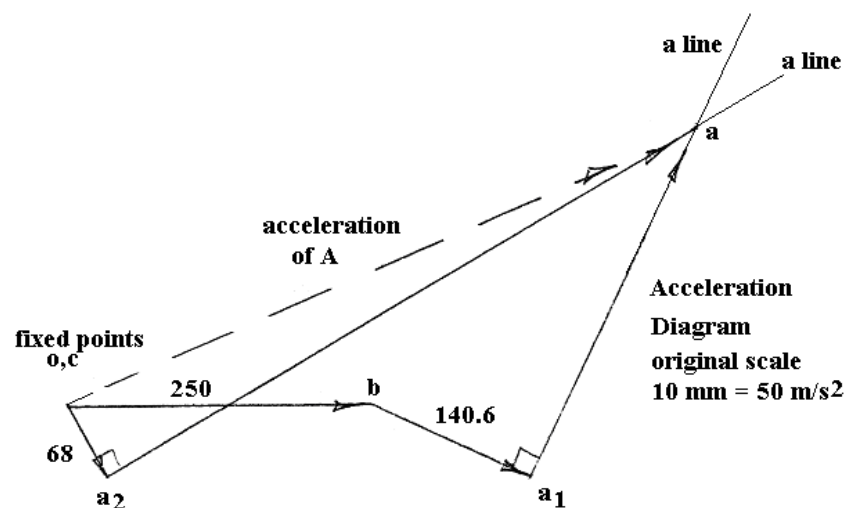
$$\text{Radial acceleration of BC} = \omega^2 \times \text{BC} = 100^2 \times 0.025 = 250 \text{ m/s}^2. \text{ \{answer (ii)\}}$$

$$\text{Radial acceleration of AB} = v^2/\text{AB} = 3.75^2/0.1 = 140.6 \text{ m/s}^2. \text{ \{answer (ii)\}}$$

$$\text{Check same answer from } \omega^2 \times \text{AB} = 37.5^2 \times 0.1 = 140.6 \text{ m/s}^2.$$

$$\text{Radial Acceleration of OA is } v^2/\text{OA} = 1.85^2/0.05 = 68.45 \text{ m/s}^2. \text{ \{answer (ii)\}}$$

Construction of the acceleration diagram gives the result shown.

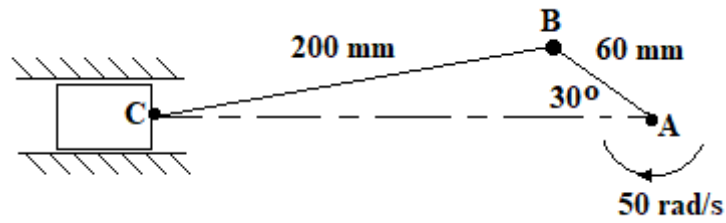


The acceleration of point A is the vector o- a shown as a dotted line. Scaling this we get  $560 \text{ m/s}^2$ .  
**\{answer (iii)\}**

### SELF ASSESSMENT EXERCISE No. 3

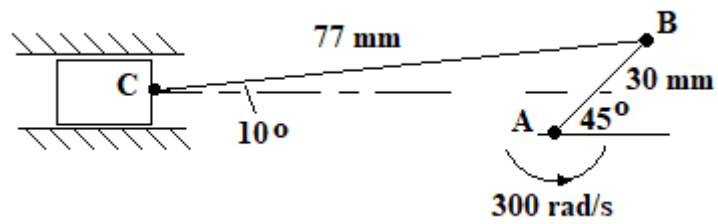
Solve the acceleration of the piston for each case shown. You should draw the space diagram out accurately first.

1.



(Ans.  $153 \text{ m/s}^2$ )

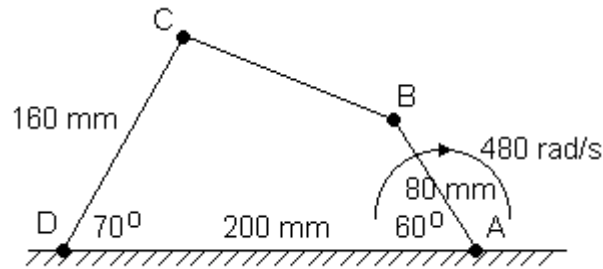
2.



(Ans.  $1\ 650 \text{ m/s}^2$ )

### WORKED EXAMPLE No. 8

Find the angular acceleration of the link CD for the case shown.

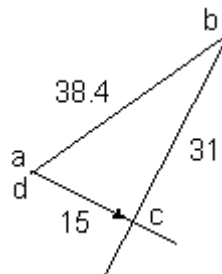


### SOLUTION

First calculate or scale the length CB and find it to be 136 mm.

Next find the velocities and construct the velocity diagram. Start with link AB as this has a known constant angular velocity.

$$(v_B)_A = \omega \times \text{radius} = 480 \times 0.08 = 38.4 \text{ m/s}$$



Next calculate all the accelerations possible.

$$\text{The centripetal acceleration of B to A is } 38.4^2/0.08 = 18\,432 \text{ m/s}^2$$

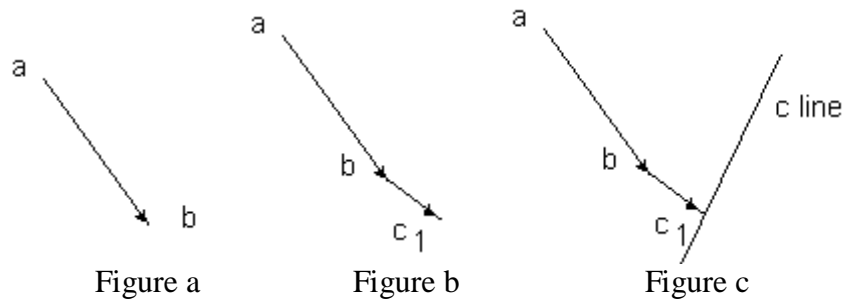
$$\text{The centripetal acceleration of C to D is } 15^2/0.16 = 1\,406 \text{ m/s}^2$$

$$\text{The centripetal acceleration of C to B is } 31^2/0.136 = 7\,066 \text{ m/s}^2.$$

We cannot calculate any tangential acceleration at this stage.

The stage by stage construction of the acceleration diagram follows.

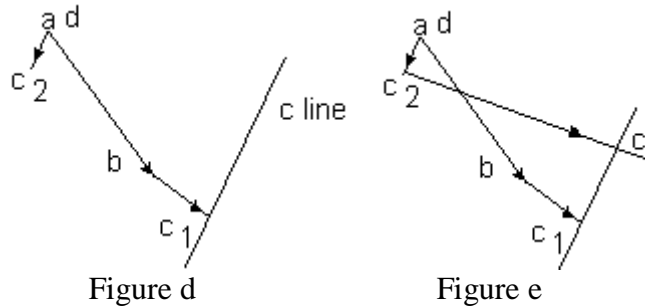
First draw the centripetal acceleration of B to A (Figure a). There is no tangential to add on).



Next add the centripetal acceleration of C to B (figure b)

Next draw the direction of the tangential acceleration of C to B of unknown length at right angles to the previous vector (figure c). Designate it as a c line.

We cannot proceed from this point unless we realise that points a and d are the same (there is no velocity or acceleration of D relative to A). Add the centripetal acceleration of C to D (figure d). This is  $1\,406\text{ m/s}^2$  in the direction of link CD. Designate it d c<sub>2</sub>.



Finally draw the tangential acceleration of C to D at right angles to the previous vector to intersect the c line (figure e).

From the diagram determine c<sub>2</sub> c to be  $24\,000\text{ m/s}^2$ . This is the tangential acceleration of C to D. The angular acceleration of the link DC is then:

$$\alpha_{(CD)} = 24000/0.16 = 150\,000\text{ rad/s}^2 \text{ in a clockwise direction.}$$

Note that although the link AB rotates at constant speed, the link CD has angular acceleration.

### WORKED EXAMPLE No. 9

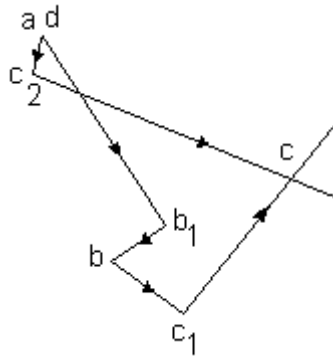
The same arrangement exists as shown for example 5 except that the link AB is decelerating at  $8000 \text{ rad/s}^2$  (i.e. in an anticlockwise direction). Determine the acceleration of the link CD.

### SOLUTION

The problem is essentially the same as example 5 except that a tangential acceleration now exists for point B relative to point A. This is found from

$$a_T = \alpha \times AB = 80000 \times 0.08 = 6400 \text{ m/s}^2$$

The direction is for an anticlockwise tangent. This is vector  $b_1 b$  which is at right angles to  $a b_1$  in the appropriate direction. The new acceleration diagram looks like this.

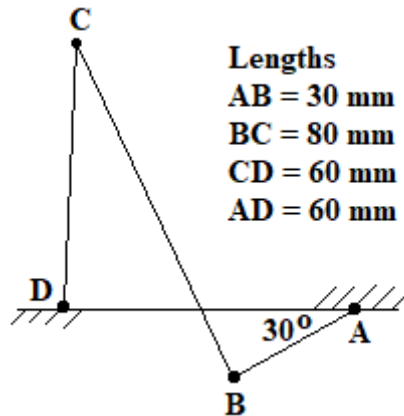


Scaling off the tangential acceleration  $c_2 c$  we get  $19300 \text{ m/s}^2$ . Converting this into the angular acceleration we get

$$\alpha = 19300/0.16 = 120625 \text{ rad/s}^2 \text{ in a clockwise direction.}$$

**SELF ASSESSMENT EXERCISE No. 4**

1. The diagram shows a 4 bar chain. The link AB rotates at a constant speed of 5 rad/s in an anticlockwise direction. For the position shown, determine the angular acceleration of the link DC.



(Answer 30 rad/s<sup>2</sup> in an anticlockwise direction)

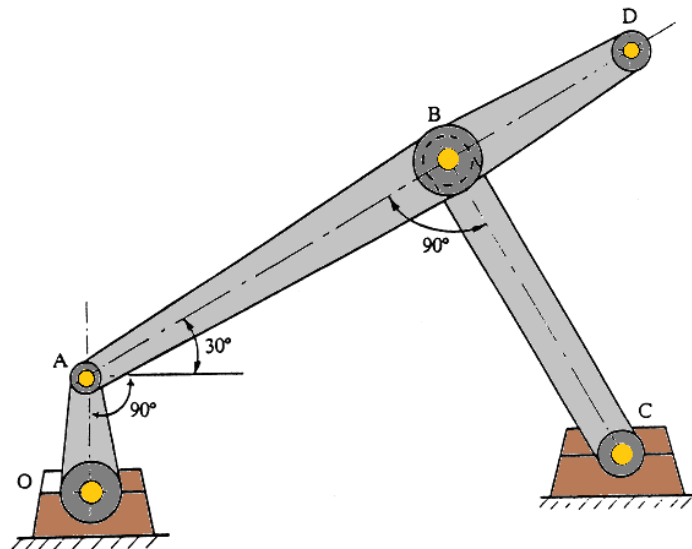
2. Repeat question 1 but this time the link AB is accelerating at 15 rad/s<sup>2</sup>.

(Answer 15.3 rad/s<sup>2</sup> in an anticlockwise direction)

3. The diagram shows the instantaneous position of a mechanism in which member OA rotates anticlockwise with an angular velocity of 100 rad/s and angular acceleration of 10 000 rad/s<sup>2</sup> in the same direction. BD is a continuation of the rigid link AB. The links have the following lengths.  
 OA – 30 mm    BC – 90 mm    AD – 168 mm    AB 1120 mm

Determine the linear the following.

- i. The velocities of points A, B and D (1.5 m/s, 2.6 m/s and 2.7 m/s)  
 ii. The absolute linear accelerations of points A and B (424.26 m/s<sup>2</sup> and 440 m/s<sup>2</sup>)



## 8. Coriolis Acceleration

Consider a link rotating at  $\omega$  rad/s and accelerating at  $\alpha$  rad/s<sup>2</sup>. On the link is a sliding element moving away from the centre of rotation at velocity  $v_R = dR/dt$  (positive if getting larger)

The link has a tangential velocity  $v_T = \omega R$

The component of this velocity in the x direction is

$$v_T \sin \theta = v_T \sin \omega R$$

The velocity  $v_R$  also has a component in the x direction

And this is  $v_R \cos \theta$

The total velocity in the x direction of the sliding link is

$$v_x = -v_T \sin \theta + v_R \cos \theta$$

$$v_x = -\left(\frac{dR}{dt}\right) \cos \omega t + \omega R \sin \omega t$$

The acceleration in the x direction is  $a_x$

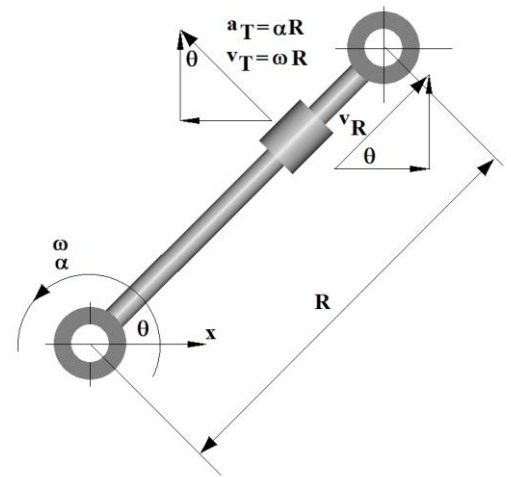
$$a_x = \frac{dv_x}{dt} = -\frac{d^2R}{dt^2} \cos(\omega t) + \frac{dR}{dt} \omega \sin(\omega t) + \omega^2 R \cos(\omega t) + \frac{dR}{dt} \omega \sin(\omega t) + \frac{d\omega}{dt} R \sin(\omega t)$$

$$a_x = -\frac{d^2R}{dt^2} \cos(\omega t) + 2 \frac{dR}{dt} \omega \sin(\omega t) + \omega^2 R \cos(\omega t) + \frac{d\omega}{dt} R \sin(\omega t)$$

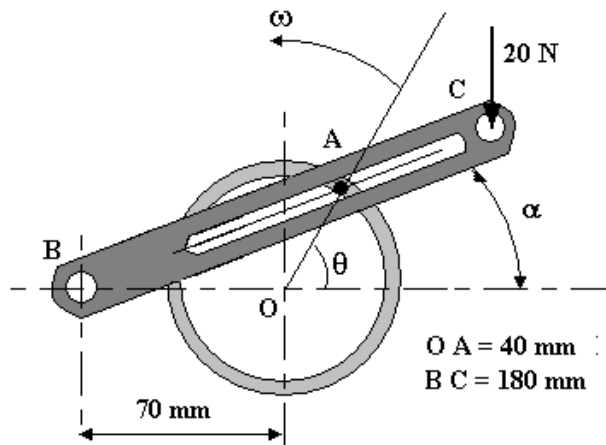
$$a_x = -\frac{d^2R}{dt^2} \cos(\theta) + 2 \frac{dR}{dt} \omega \sin(\theta) + \omega^2 R \cos(\theta) + \alpha R \sin(\theta)$$

When  $\theta = 90^\circ$

$$a_x = a_T = 2\omega \frac{dR}{dt} + \alpha R = 2\omega v_R + \alpha R$$



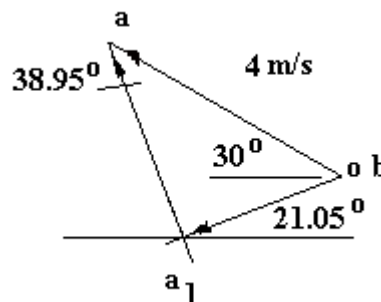
### WORKED EXAMPLE No. 10



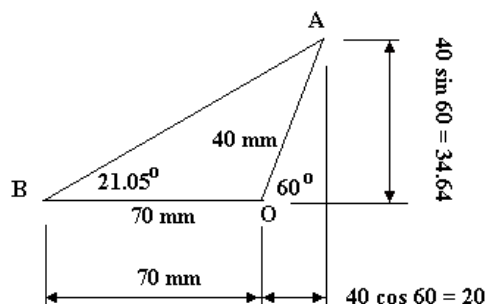
The diagram shows part of a quick return mechanism. The pin A slides in the slot when the disc is rotated. Calculate the angular velocity and acceleration of link BC when  $\theta = 60^\circ$  and  $\omega = 100 \text{ rad/s}$ .

### SOLUTION

The tangential velocity of A relative to O is  $\omega R = 100 \times 0.04 = 4 \text{ m/s}$ . The velocity diagram is constructed as shown.



The tangential velocity of pin A relative to B is  $(V_{A1})_B = a - a_1 = 4 \cos(38.95^\circ) = 3.11 \text{ m/s}$   
 The radial velocity of A relative to B is  $(V_A)_B = 4 \sin(38.95^\circ) = 2.515 \text{ m/s}$   
 The length of BA is easily calculated from the diagram.



$$BA = \sqrt{(34.64^2 + 90^2)} = 96.44 \text{ mm}$$

The angular velocity link BC =  $3.11/BA = 32.2 \text{ rad/s}$



### Analytical Method

The angle of link BC is

$$\alpha = \tan^{-1} \left[ \frac{40 \sin \theta}{70 + \cos \theta} \right] = \tan^{-1} \left[ \frac{\sin \theta}{7/4 + \cos \theta} \right]$$

The angular velocity is  $d\alpha/dt$  and the tools for doing the differentiation are normally given in the question as follows.

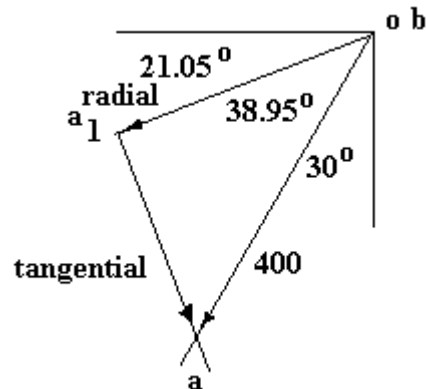
$$x = \left[ \frac{\sin \theta}{7/4 + \cos \theta} \right] \quad \frac{d\alpha}{dx} = \frac{1}{1+x^2} = \frac{1}{1 + \left( \frac{\sin \theta}{7/4 + \cos \theta} \right)^2} \quad \frac{dx}{d\theta} = \frac{1 + 7/4 + \cos \theta}{(7/4 + \cos \theta)^2}$$

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dx} \times \frac{dx}{d\theta} = \frac{1}{1 + \left( \frac{\sin \theta}{7/4 + \cos \theta} \right)^2} \times \frac{1 + 7/4 + \cos \theta}{(7/4 + \cos \theta)^2}$$

Put  $\theta = 60^\circ$  and evaluate and  $d\alpha/d\theta = 0.316$

$\theta = \omega t$  so  $d\theta = \omega dt$   $dt = d\theta/\omega$  so  $d\alpha/dt = 0.316 \times \omega = 31.6 \text{ rad/s}$  which is close to the answer found before.

Next construct the acceleration diagram.



Link O A only has centripetal acceleration inwards  $(a_A)_O = \omega^2 R = 100^2 \times 0.04 = 400 \text{ m/s}^2$

The pin A has a tangential acceleration and Coriolis acceleration normal to the link. It has centripetal acceleration and radial acceleration towards the centre of rotation. B. The diagram can be constructed without calculating them.

The Coriolis acceleration is  $2 \omega v$  where  $\omega = 32.2$  and  $v$  is the radial velocity = 2.515 m/s

The Coriolis term is hence  $161.97 \text{ m/s}^2$

The tangential acceleration of A relative to B is

$$a_1 - a = 400 \sin 38.95 = 251.46 \text{ m/s}^2$$

Part of this is the Coriolis so the tangential acceleration is  $251.46 - 161.97 = 89.49 \text{ m/s}^2$

The angular acceleration of link AC is  $\alpha = 89.49 / BA = 89.49 / 0.09644 = 928 \text{ rad/s}^2$

The direction is negative (clockwise) so it is decelerating.

### SELF ASSESSMENT EXERCISE No. 5

A link OA is 80 mm long and rotates at a constant speed of 50 rad/s. A sliding link attached to it slides on link BC and makes BC rotate about B as shown. Calculate the angular velocity and acceleration of BC when angle  $\theta = 70^\circ$ .

(22.8 rad/s and  $-76.2 \text{ rad/s}^2$ )

