D225 2005 Q6

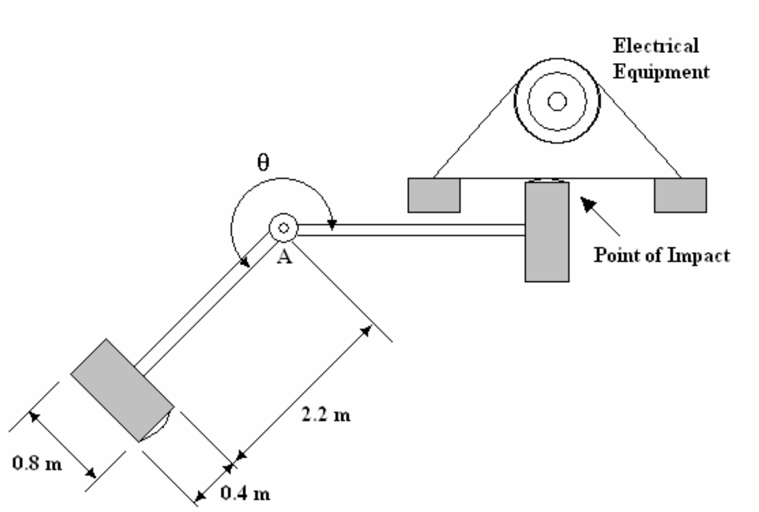
The diagram shows a swinging hammer used for shock-testing electrical equipment intended for Naval applications. The hammer comprises a relatively thin uniform bar of length 2.2 m and mass 150 kg and a mass of 250 kg fixed at the end of the bar as shown. The uniform bar is pivoted at point A and the 250 kg mass at the end is uniformly distributed over the area 0.8 m x 0.4 m.

a) Determine the moment of inertia at the hammer about the pivot A.

b) Assuming zero friction at the pivot and that the hammer falls freely from rest at an initial position $\theta = 95°$, express the angular velocity, $\dot{\theta}$, in terms of the angle, θ.

c) The electrical equipment has a mass of 140 kg and it is supported on brackets until the hammer strikes it. Assuming that the electrical equipment behaves as a single rigid mass and that the impact is perfectly elastic, calculate the vertical velocity of the electrical equipment and the angular velocity of the hammer immediately after the impact.

d) If the tip of the hammer has an effective stiffness of 200 MN/m, estimate the peak acceleration experienced by the electrical equipment.



a) For a uniform rod $I = \rho A \int x^2 dx$ where x is the distance from the pivot, A is the cross sectional Area and ρ is the density.

For the rod $I = \rho A \int_0^{2.2} x^2 dx = \rho A \left[\frac{x^3}{3}\right]_0^{2.2} = \frac{\rho A x(x^2)}{3} = M\frac{x^2}{3} = 150\frac{2.2^2}{3} = 242 \text{ kg m}^2$

Check for uniform rod k = 0.5773L = 1.27m $I = Mk^2 = 242 \text{ kg m}^2$

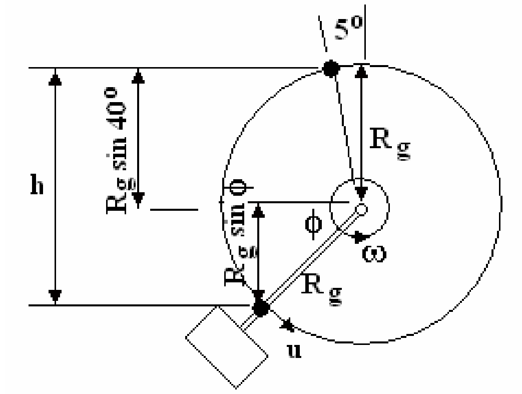
For the hammer $I = \rho A \int_{2.2}^{2.6} x^2 dx = \rho A \left[\frac{x^3}{3}\right]_{2.2}^{2.6} = \frac{\rho A(2.6^3 - 2.2^3)}{3}$ =2.30933ρA

The mass is 0.4ρA so $I = M$ (2.30933/0.4) = 250(2.30933/0.4) = 1443.33 kg m^2

Rough check k ≈2.4 $I = Mk^2 = 1440 \text{ kg m}^2$

Total I = 242 + 1443.33 = 1685.33 kg m^2

Radius of gyration $I = Mk^2$ k = √I/M = √(1685.33 /400) = 2.052 m b). Taking the change in height from the top to the position shown :



Potential energy change = Mg h

$h = R_g \sin 85^o + R_g \sin \phi$

PE = Mgh = 400g $R_g(\sin 85^o + \sin\phi)$ = 400g x 1.9125$(\sin 85^o + \sin\phi) = 7504.65(\sin 85^o + \sin\phi)$

The kinetic energy change (assumed zero at the maximum height) is $I\,\omega^2/2 = 1685.33\,\omega^2/2 = 842.665\,\omega^2$

Equating energy $842.665\,\omega^2 = 7504.65(1+\sin\phi)$

$$\omega = \sqrt{\frac{7504.65(\sin 85^o + \sin\phi)}{842.665}} = 2.984\sqrt{\sin 85^o + \sin\phi}$$

In terms of the angle θ, $\phi = \theta - 180$ and $\sin(\theta - 180) = -\sin\theta$ so $\sin\phi = -\sin\theta$

$$\omega = 2.984\sqrt{\sin 85^o - \sin\theta}$$

c) Just before impact the hammer has swung to $\theta = 360^o$

$$\omega = 2.984\sqrt{\sin 85^o - \sin 360} = 2.978 \text{ rad/s}$$

Initial angular momentum of the hammer = I ω = 1685.33 x 2.978 = 5018.9 kg m^2/s

The equivalent linear momentum is found by dividing by radius.

Momentum = Iω/R and since ω = v/R the linear momentum is Iv/R^2

At the point of contact this is 2.4 m

At the point of impact u = ωR = 2.978 x 2.4 = 7.15 m/s

Initial linear momentum is 1685.33 x $7.15/2.4^2$ = 2092 kg m/s

Initial momentum of Equipment = 0

Initial total momentum = 2092 kg m/s

Relative velocity = $u_1 - u_2$ = 7.15 m/s (closing)

Final momentum at the instant after impact = $I\,v_1/R^2 + 140\,v_2$

Final momentum = $1685.33\,v_1/2.4^2 + 140\,v_2$

Final momentum = initial momentum =2092 = $292.6\,v_1 + 140\,v_2$ ....(A)

v_1 = velocity of the hammer and v_2 = velocity of the equipment.

For a perfectly elastic collision the relative velocity after the collision is

$7.15 = -(v_1 - v_2) = -v_1 + v_2$

$v_1 = v_2 - 7.15$

Substitute into (A)

$2092 = 292.6\,(v_2 - 7.15) + 140(v_2)$

$2092 = 292.6\,v_2 - 2092 + 140\,v_2$

$4184 = 432.6\,v_1$

v_2 = 9.67 m/s

v_1 = 9.67 - 7.15 = 2.52 m/s

Check final momentum = 292.6 x 2.52 +140 x 9.67 = 2092

Angular velocity of the hammer after impact is ω = v_1/Radius = 2.52/2.4 = 1.05 rad/s d)

Change in momentum of equipment = mv = 140 x 9.67 = 1353.8 kg m/s
Impulse applied = F Δt
These equate so F = 1353.8/Δt

Change in angular momentum of the hammer = I ($\omega_1 - \omega_2$) = 1685.33(2.978 – 1.05) = 3249.3 kg m^2/s
Angular impulse = FR Δt = 2.4 F Δt
These Equate so F = 1353.8/Δt
As the forces should be equal (and opposite) this is no surprise.

I can see no way to find F as this is needed to find the acceleration and I cannot see how the elastic stiffness of the tip can be used to find it.