## D225 2005 Q3

The diagram shows a mechanism comprising of a rigid bar OY pivoted about the fixed point O . A spring is attached at point Y on the bar which exerts a vertical force of 8000 h Newtons. The disc rotates clockwise about the centre C and a peg X fixed on the disc slides without friction in a slot. A point mass of 2.5 kg is present at Y and all other masses are negligible. The relevant lengths are given. At one instant the angle $\angle \mathrm{OCX}$ is $20^{\circ}$ and the disc is rotating at a constant velocity of $120 \mathrm{rad} / \mathrm{s}$ (clockwise)
(a) Calculate the positions of points X and Y at this instant and the downward force in the spring.
(b) Find the velocity vectors for points X and Y at this instant and the kinetic energy present at the point mass attached at Y.
(c) Determine the angular acceleration of the bar OY at this instant.
(d) Establish the torque which must be exerted on the disc at this instant to ensure that the disc does not change its angular velocity.

(a)
$\mathrm{BX}=80 \sin 20=27.362$
$\mathrm{BC}=80 \cos 20=75.175$
$\mathrm{OB}=250-75.175=174.825$
$\theta=\tan ^{-1}(27.362 / 174.825)=8.895^{\circ}$
$\mathrm{h}=400 \sin 8.895^{\circ}=61.851 \mathrm{~mm}$
Force $=8000 \mathrm{~h}=8000 \mathrm{x} 0.061851=371.1 \mathrm{~N}$
$\mathrm{OX}=\sqrt{ }\left(174.825^{2}+27.362^{2}\right)=176.95 \mathrm{~mm}$

(b) $\omega=120 \mathrm{rad} / \mathrm{s}$

The tangential velocity of X relative to O is $\omega \mathrm{R}=120 \times 0.08=9.6 \mathrm{~m} / \mathrm{s}$. The velocity diagram is constructed as shown.

The tangential velocity of pin X relative to O is
$\left(\mathrm{V}_{\mathrm{x}}\right)_{\mathrm{ol}}=9.6 \sin (70-8.9)=8.4 \mathrm{~m} / \mathrm{s}$
The tangential velocity of point Y is proportional to the radius from O so (Vy)o $=8.4 \times 400 / 176.95=19 \mathrm{~m} / \mathrm{s}$
The kinetic energy $=1 / 2\left(M v^{2}\right)=1 / 2\left(2.5 \times 19^{2}\right)=451.25 \mathrm{~J}$


The radial velocity of X relative to O is $\mathrm{o}-\mathrm{o} 1=\sqrt{ }\left(9.6^{2}-8.4^{2}\right)=4.648 \mathrm{~m} / \mathrm{s}$
The tangential veolocity of the bar OY is $\omega=$ tangential velocity/radius
$\omega=19 / 0.4=47.5 \mathrm{rad} / \mathrm{s}$
Check $\omega=8.4 / .17695=47.5 \mathrm{rad} / \mathrm{s}$
The pin X has a tangential acceleration and Coriolis acceleration normal to the link. It has centripetal acceleration and radial acceleration towards the centre of rotation $O$. The diagram can be constructed without calculating them.
The Coriolis acceleration is $2 \omega \mathrm{v}$ where $\omega=47.5$ and v is the radial velocity $=4.648 \mathrm{~m} / \mathrm{s}$
The Coriolis term is hence $220.77 \mathrm{~m} / \mathrm{s}^{2}$
The centripetal acceleration of X to C is $\mathrm{v}^{2} / \mathrm{r}$ or $\omega^{2} \mathrm{r}=120^{2} \times 0.080=1152 \mathrm{~m} / \mathrm{s}^{2}$

The acceleration diagram can be drawn.
The tangential acceleration of x relative to O is
$\mathrm{x}_{1}-\mathrm{x}=1152 \sin 28.9^{\circ}==556.7 \mathrm{~m} / \mathrm{s}^{2}$
Part of this is the Coriolis so the tangential acceleration is

$556.7-220.7=336 \mathrm{~m} / \mathrm{s}^{2}$
The angular acceleration of link AC is $\alpha=336 / \mathrm{OX}=336 / 0.08=4200 \mathrm{rad} / \mathrm{s}^{2}$
The direction is negative (clockwise) so it is accelerating.
The tangential acceleration of $Y$ relative to $O$ is $556.7 \times 400 / 176.95=1258.4 \mathrm{~m} / \mathrm{s}^{2}$
The inertia force is $\mathrm{Fi}=\mathrm{M} \mathrm{a}=2.5 \times 1258.4=3146 \mathrm{~N}$ and acts normal to the link.
Resolve the spring force normal to the link. Fs $=371.1 \cos 8.9^{\circ}=366.6 \mathrm{~N}$
The total force at Y acting normal to the link is $3146-366.6=2779.4 \mathrm{~N}$
The force acting normal to the link at X is $2779.4 \times 400 / 176.95=$ 6283 N
Resolve this normal to CX
$\mathrm{F}_{\mathrm{n}}=6283 \sin 61.1^{\circ}=5500 \mathrm{~N}$
Torque produced on disc $=5500 \times 0.08=440 \mathrm{Nm}$

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