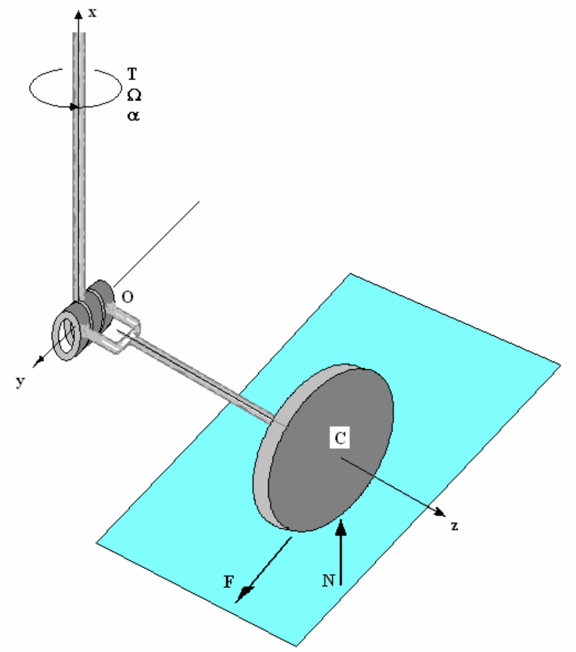
D225 2005 Q2

The diagram shows a wheel centre C rolling on a horizontal surface. The wheel can spin freely about the horizontal shaft OC. Shaft OC is connected to at O to a vertical shaft OX though a clevis which acts a frictionless horizontal pivot.

The angular velocity of the vertical shaft is Ω and the angular acceleration is α. The torque applied to the vertical shaft is T. The axis set Oxyz moves with the wheel. The mass of the wheel is M and it has a polar moment of inertial J_P and diametral inertia J_D. All other elements of the system are assumed to have no mass. The radius of the wheel is Rand the length OC is 4R.

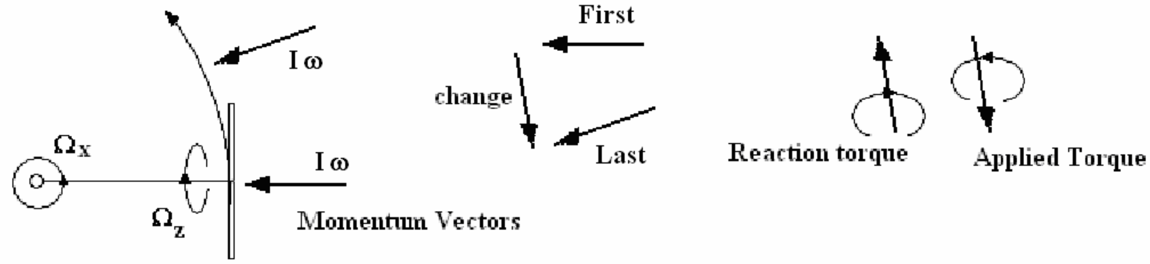


a) Assuming that the wheel is rolling without any slip, derive expressions for the torque T, the normal force N and the friction force F in terms of Ω,α,M, J_P and J_D.

b) Given the coefficient of friction μ, for the contact between the wheel and the horizontal surface on which it rolls, produce an expression for the maximum torque T, which may be applied before slip occurs.

SOLUTION

The angular velocity and acceleration of the wheel is Ω_z and is related to Ω_x.



Linear velocity of C about O is Ω_x 4R and this must be the same as the linear velocity of a point on the surface of the wheel Ω_z R.

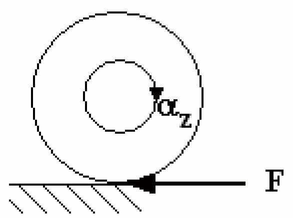
Equating $\Omega_z = 4\Omega_x$ i.e. the wheel rotates 4 revolutions for every one revolution about OX

It follows that $\alpha_z = 4\alpha_x$

The spinning of the wheel will produce a gyroscopic couple of magnitude $J_P\Omega_x \Omega_z = 4J_P\Omega_x^2$

The reaction torque is deduced as shown showing that the wheel will tend to push down onto the surface.

The Normal force N is the sum of the weight and the force due to the reaction torque.



$N = Mg + 4J_P\Omega_x^2/4R$

$N = Mg + J_P\Omega_x^2/R$

The acceleration of the wheel about C is $\alpha_z = 4\alpha_x$

The torque to accelerate the wheel about C is $T_z = J_P\ \alpha_z= 4J_P\ \alpha_x$

The force applied by the surface to produce this is F

$F= T_z /R = 4J_P\ \alpha_x/R$ $\qquad$ $F = 4J_P\ \alpha_x/R$

This produces a torque about OX of $T = 4RF = 4R(4J_P\ \alpha_x/R) = 16J_P\ \alpha_x$

The acceleration about the axis OX also requires an inertia torque $T = J_{ox}\ \alpha_x$
J_{ox} is found with the parallel axis theorem

$J_{ox} = Mk^2 + J_D \quad k = 4R \qquad J_{ox} = 16MR^2 + J_D$
$T = \alpha\{16MR^2 + J_D\}$

The total torque about OX is $T_x = 16J_P\ \alpha_x + \alpha_x\{16MR^2 + J_D\}$

$T_x = \alpha_x\ [16J_P + 16MR^2 + J_D]$

The wheel will slip when $F = \mu N$

$F = \mu(Mg - J_P\Omega_x^2/R)$

$T = FR = \mu(Mg - J_P\Omega_x^2/R)R = \mu(MRg - J_P\Omega_x^2)$ due to friction force only
To this must be added the torque $\alpha_x\{16MR^2 + J_D\}$

$T_x = \mu(MRg + J_P\Omega_x^2) + \alpha_x\{16MR^2 + J_D\}$

This is the torque when slip occurs assuming that acceleration still occurs about OX.

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