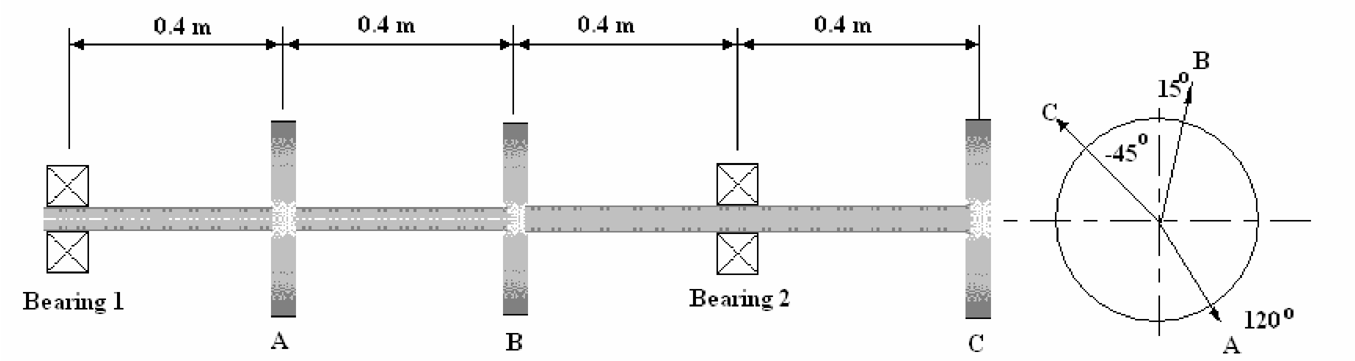
D225 DYNAMICS OF MECHANICAL SYSTEMS
2005 Q1

The diagram shows a rotor lying on the z axis whose total length is 1.6 m. The rotor is supported on stiff bearings a z = 0 and at z = 1.2 m. Three discs A,B and C are fixed to the rotor and the mass centre of each is slightly eccentric. The table shows the position of the discs and details of the imbalance carried initially on each of the discs.



The angular positions on the diagram have been added and were not on the original question.

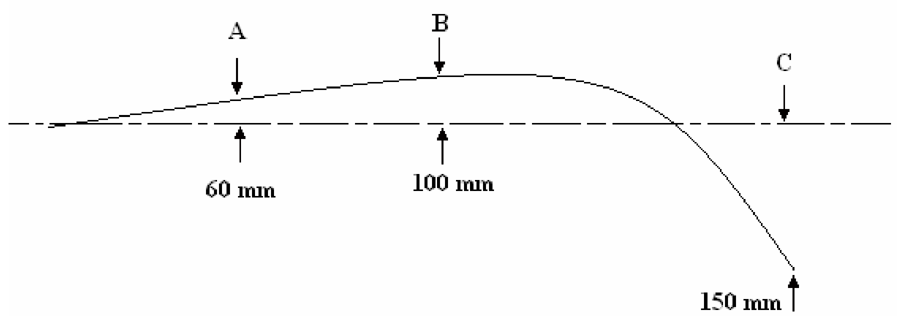
	z (m)	M (kg)	eccentricity	θ (degrees)
A	0.4	10	0.15 mm	120
B	0.8	50	0.1 mm	15
C	0.1.6	20	0.2 mm	-45

(a) Compute the magnitudes of bearing reaction forces when the rotor is spinning at 800 rev/min assuming the rotor is rigid.

(b) Compute the relative phase of the bearing reaction forces stating clearly which bearing has the leading phase. The direction of shaft rotation is such that the angles given are positive.

(c) Determine the appropriate imbalance corrections to be applied at discs A and C.

(d) Suppose that the rotor is not rigid and has a first critical speed at 4000 rev/min with a mode-shape similar to that shown. If the imbalanced rotor is spun at 3990 rev/min, comment on the (relative) sizes of the bearing reaction forces (The deflections shown are are for the mass-normalised mode).



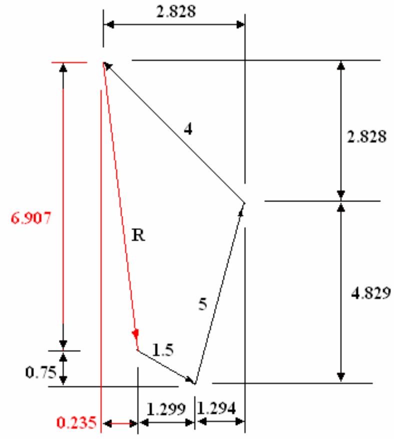
SOLUTION

Make Bearing 1 the reference plane and re-tabulate as follows.

	z (m)	M kg	R mm	MR (kg mm)	θ (degrees)	MRz (kg mm^2)
A	0.4	10	0.15	1.5	120	600
B	0.8	50	0.1	5.0	15	4000
C	1.6	20	0.2	4.0	-45	5400

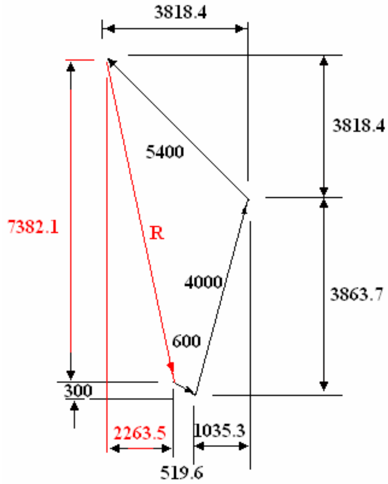
Construct the MRx polygon. The diagram is not to scale and the lengths have been calculated

$R = (2263.5^2 + 7382.1^2)^{1/2} = 7721.3$ kg mm^2
$\theta = \tan^{-1}(2263.5/7382.1) = 17^o$
Angle of the reaction is 163^o
$\omega = 2\pi N/60 = 2\pi(800/60) = 83.77$ rad/s
The out of balance moment is $\omega^2 \times 7721.3 \times 10^{-6} = 54.19$ N m based on bearing 1.



The force on bearing 2 that produces an equal and opposite couple is 54.19/0.12 = 451.6 N The angle is 180^o to the other so it is -17^o and this leads bearing A

Wee now need to draw the MR polygon.



$R = (6.907^2 + 0.235^2)^{1/2} = 6.911$ kg mm

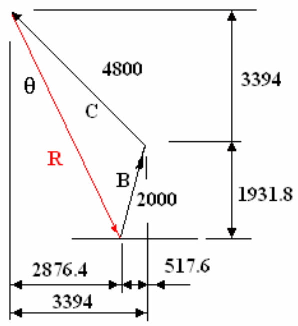
The out of balance force is $6.91\times10^{-3} \times 83.77^2$
= 48.5 N

Part c. Take A as the reference plane

	z (m)	M kg	R mm	MR (kg mm)	θ (degrees)	MRz (kg mm^2)
A	0	10	0.15	1.5	120	0
B	0.4	50	0.1	5.0	15	2000
C	1.2	20	0.2	4.0	-45	4800

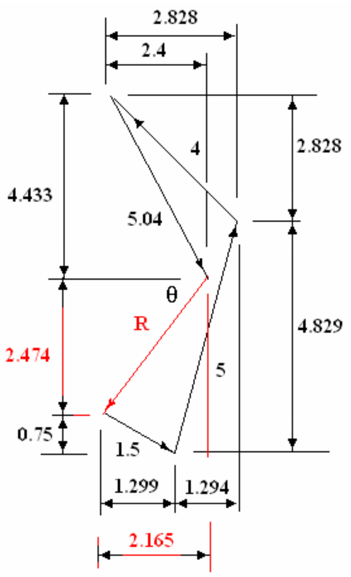
$R = (2876^2 + 5325.8^2)^{1/2} = 6053$ kg mm^2

$\theta = \tan^{-1}(2876.4/5325.8) = 28.4^o$
This is 151.6^o from the ref. direction.
If this is to be added to C then the MR needed = 6053/1200 = 5.04 kg mm



For static balance we need the MR polygon

From this we find $R = (2.474^2 + 2.165^2)^{1/2} = 3.29$ kg mm
$\theta = \tan^{-1}(2.474/2.165) = 48.8^o$
This is 221.2^o from the ref direction



Part d

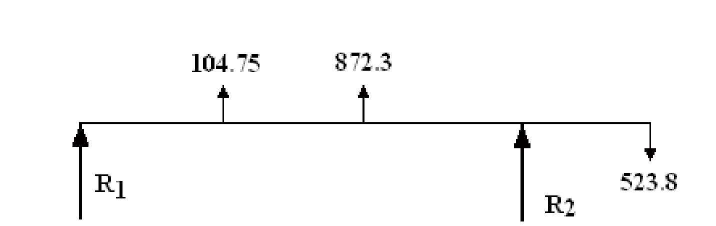
Hard to understand what is required here? What is the relevance of the critical speed? Clearly the massive deflections will produce much larger centrifugal forces that will swamp the initial out of balance forces. These will presumably be in one plane so the problem is similar to a simply supported beam.

The forces are $M\omega^2R$ $\omega = 3990 \times 2\pi/60 = 417.8$ rad/s

$F_A = 10\,(417.8)^2(0.06) = 104.75$ kN

$F_B = 50\,(417.8)^2(0.1) = 872.3$ kN

$F_C = 20\,(417.8)^2(0.15) = 523.8$ kN



Taking moments about R_1

$104.75 \times 0.4 + 872.3 \times 0.8 - 523.8 \times 1.6 = R_2 \times 1.2$

$41.9 + 697.84 - 838.08 = -98.34$

$R_2 = -81.95$ kN

$R_1 + 872.3 + 104.75 = 523.8 + 81.95$

$R_1 = -371.3$ kN