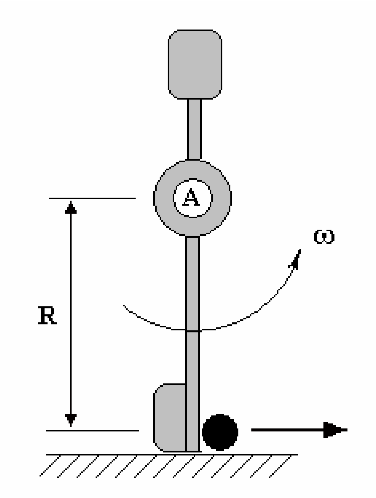
<u>D225 DYNAMICS OF MECHANICAL SYSTEMS Q9 2004</u>

The diagram shows the rotating part of a "machine-gun" for firing squash balls. The rotor which is pivoted at A has inertia of J_p and it rotates at an average angular velocity of ω rad/s. The working radius of the rotor is R and the rotor may be treated as being rigid and being mounted on rigid bearings for the purposes of an initial analysis. The effective coefficient of restitution for impacts between the rotor and each squash ball is η and the mass of one squash ball is m.
Develop expressions for:



(a) The change in the rotation speed of the rotor between the start of any impact with a ball and the end of that impact.
(b) The speed at which squash balls are shot forward from the gun.
(c) The power consumed if one squash ball is discharged in each revolution of the rotor and if there are no other power losses in the system.

SOLUTION
WARNING – I am not sure about the following theory for effective coefficient of restitution. Everyone defines it differently. The common element is the impulse.

Linear Impulse $= F\,\Delta t$
Change in momentum of ball $= m(v - 0) = mv$
Impulse = change in momentum $\quad F\,\Delta t = mv \quad \Delta t = mv/F$

Angular impulse $= T\,\Delta t \quad T = FR \quad$ Angular impulse $= FR\,\Delta t$
Change in Angular Momentum $= I(\omega_1 - \omega_2)$
Impulse = change in momentum $\quad FR\,\Delta t = I(\omega_1 - \omega_2) \quad \Delta t = I(\omega_1 - \omega_2)/FR$

Equate for time Δt $\quad I(\omega_1 - \omega_2)/FR = mv/F \quad (\omega_1 - \omega_2) = mvR/I$ …………..(1)

Scratching my head for a meaning to "effective coefficient of restitution" a logical definition might be

η = relative velocity before collision/relative velocity after collision. This normally applies to the collision of two linear objects so I will use the linear velocity of the bat at the point of contact.
Velocity of bat before collision $= \omega_1 R \quad$ Velocity of ball = 0
Relative velocity before collision $= \omega_1 R$
Velocity of bat after collision $= \omega_2 R \quad$ Velocity of ball = v
Relative velocity before collision $= \omega_2 R - v$

$$\eta = \frac{\omega_2 R - v}{\omega_1 R} \qquad \eta\omega_1 R = \omega_2 R - v$$

The average velocity is $\Omega = (\omega_1 + \omega_2)/2$
$\omega_1 = \Omega + \frac{1}{2}(\omega_1 - \omega_2) \qquad \omega_2 = \Omega - \frac{1}{2}(\omega_1 - \omega_2)$

$$\eta R\left\{\Omega + \frac{\omega_1 - \omega_2}{2}\right\} = R\left\{\Omega - \frac{\omega_1 - \omega_2}{2}\right\} - v$$

substitute (1) into this.

$$\eta R\left\{\Omega + \frac{mvR}{2I}\right\} = R\left\{\Omega - \frac{mvR}{2I}\right\} - v \qquad v\left\{\frac{\eta m R^2}{2I} + \frac{mR^2}{2I} + 1\right\} = R\Omega(1-\eta)$$

$$v = \frac{R\Omega(1-\eta)}{1 + \frac{mR^2}{2I}(1+\eta)}$$

POWER

Kinetic energy before collision = I $\omega_1^2/2$
Kinetic energy after collision = I $\omega_2^2/2 + mv^2/2$
Loss of kinetic energy = I $\omega_1^2/2 - (I\,\omega_2^2/2 + mv^2/2) = ½\{I(\omega_1^2 - \omega_2^2) - mv^2\}$
This occurs in one revolution so time taken to restore the energy is $t = 2\pi/\Omega$

$$\text{Power} = \frac{\Omega\left\{I\left(\omega_1^2 - \omega_2^2\right) - mv^2\right\}}{4\pi} = \frac{\Omega\left\{I\left(\omega_1 - \omega_2\right)\left(\omega_1 + \omega_2\right) - mv^2\right\}}{4\pi} \text{ and since} = \frac{\omega_1 + \omega_2}{2} = \Omega \text{ (average)}$$

$$P = \frac{\left\{I\Omega^2\left(\omega_1 - \omega_2\right) - mv^2\Omega\right\}}{2\pi}$$

substitute $(\omega_1 - \omega_2) = mvR/I$

$$P = \frac{\Omega^2 mvR - mv^2\Omega}{2\pi} = -\frac{mv\Omega\{\Omega R + v\}}{2\pi}$$