<u>DYNAMICS OF MECHANICAL SYSTEMS D225 Q8</u>

A builder of model cars has decided to drive them with permanent magnet motors and he is eager to obtain the maximum acceleration possible from his models. The total mass of his model car is 600 g. The motor has a maximum torque capability of 0.06 Nm and it has an inertia of 1×10^{-5} kg m^2.

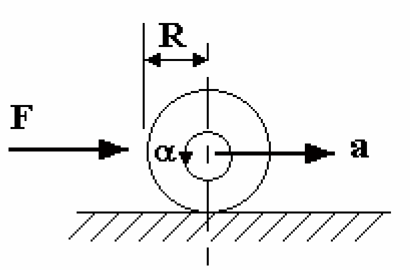
(a) Calculate the optimum gearing ratio (measured in mm of linear travel per revolution of the motor).

(b) Rolling losses in the model cars are quite high. At a velocity of 3 m/s, the total rolling losses are equivalent to a 0.5 N drag force on the car and for velocities above this one, the equivalent drag force rises in proportion to the square of the velocity.

Calculate the maximum velocity of the car based on the gearing ratio you have chosen.

<u>SOLUTION</u>

(a) Idealise the model as a wheel as shown. Mass = m Radius = R



Linear acceleration = a   Angular acceleration = α

The total torque from the motor is the Torque required to accelerate the wheel plus the torque to produce linear acceleration.

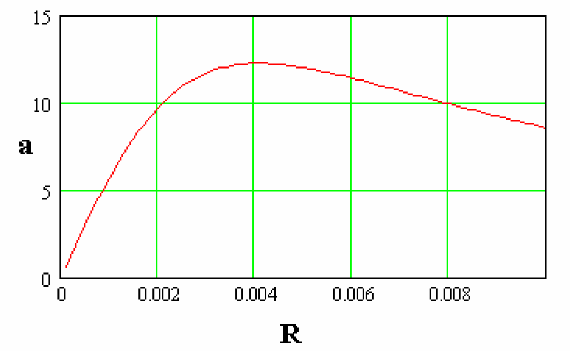
$T = I\alpha + FR$ where $F = ma$

$T = I\alpha + maR$ but $\alpha = a/R$

$T = I\,a/R + maR = a(I/R + mR)$

$a = T/(I/R + mR) = TR/(I + mR^2)$

For the benefit of students a plot of the function is shown.



To find the radius that produces maximum acceleration use max and mi theory. Differentiate the function with respect to R and equate to zero.

Differentiating reveals

$$\frac{da}{dR} = \frac{T}{I + mR^2} - 2mR^2 \frac{T}{(I + mR^2)^2} = 0$$

$$\frac{T}{I + mR^2} = 2mR^2 \frac{T}{(I + mR^2)^2} \qquad 1 = 2\frac{mR^2}{(I + mR^2)} \qquad I + mR^2 = 2mR^2$$

$$I = 2mR^2 - mR^2 = mR^2 \qquad R = \sqrt{\frac{I}{m}}$$

Evaluate and $R = \sqrt{(1 \times 10^{-5}/0.6)} = 4.082 \times 10^{-3}$ m or 4.082 mm (confirmed by the graph)

Gear ratio G = circumference/1 = $2\pi R$ = 25.65 mm per revolution.

The optimal acceleration is

$$a = \frac{TR}{I + mR^2} = \frac{0.06 \times 4.082 \times 10^{-3}}{1 \times 10^{-5} + 0.6 \times (4.082 \times 10^{-3})^2} = 12.247 \text{ m/s}^2$$

(b) It is hard to make sense of the information given. I presume the following

$D = cv^2$ When $v = 3$ m/s $D = 0.5$ N so $c = 0.5/9 = 0.05555$ and so $D = 0.05555\ v^2$

When the velocity reaches its maximum, the acceleration is zero and all the motor torque goes into overcoming drag.

$$F = ma + D = ma + cv^2$$
$$T = I\alpha + FR = I\alpha + \left(ma + cv^2\right)R$$
$$T = I\frac{a}{R} + \left(ma + cv^2\right)R \quad \text{put } a = 0$$
$T = cv^2R$ $\quad v^2 = T/cR = 0.06/(0.05555 \times 4.082 \times 10^{-3}) = 26.46 \quad v = 5.144$ m/s