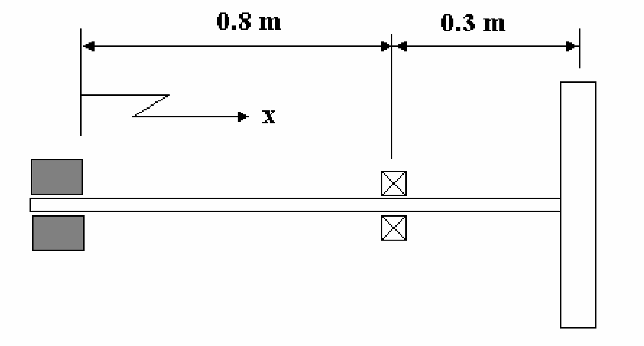
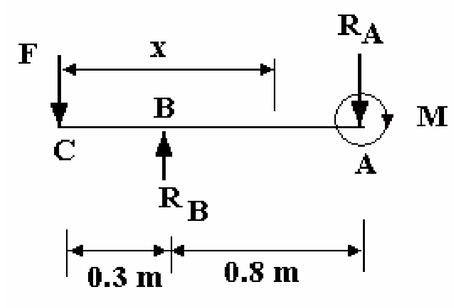
The diagram shows a rotor suspended in two bearings. The bearing at the left hand end keeps the shaft perfectly horizontal at all points to the left of x = 0. It also prevents the rotor from translation normal to its axis. The bearing at the right hand side (x = 0.8 m) prevents translation of the rotor shaft normal to the axis at this point but it does not oppose rotation. A solid steel disc of outside diameter 0.8 m and axial thickness 0.05 m is fixed on the end of the shaft. The diameter of the steel shaft is 40 mm and Young's modulus for the shaft steel is 200 GPa. The density of the steel used for both the shaft and the overhung disc is 7800 kg/m^3.



Estimate the first natural frequency of this system using a Rayleigh quotient approximation.

SOLUTION

First draw the simplified force system. M is the moment at the fixed bearing. It is advisable to draw the diagram with the fixed end on the right so that M is excluded from the formula.



Next calculate M and F

M = Mass of disc = $\pi(D^2/4)$ x t x ρ = 195.92 kg

Weight = F = mg =1922 N

Set up Macaulay's equation.

$$M_x = EI\frac{d^2y}{dx^2} = -Fx + R_B[x-0.3]$$

$$EI\frac{dy}{dx} = -F\frac{x^2}{2} + R_B\frac{[x-0.3]^2}{2} + A$$

$$EIy = -F\frac{x^3}{6} + R_B\frac{[x-0.3]^3}{6} + Ax + B$$

Use the known boundary conditions to solve. Remember any square bracket that becomes negative is ignored.

When x =1.1 dy/dx = 0

$$0 = -F\frac{1.1^2}{2} + R_B\frac{[0.8]^2}{2} + A \qquad A = 0.605F - 0.32R_B$$

When x =1.1 y = 0

$$0 = -F\frac{1.1^3}{6} + R_B\frac{[0.8]^3}{6} + (0.605F - 0.32R_B)1.1 + B$$

$$0 = -0.2218F + 0.0853R_B + 0.6655F - 0.352R_B + B \qquad B = -0.4437F + 0.2667R_B$$

When x = 0.3 y = 0

$$0 = -F\frac{0.3^3}{6} + R_B\frac{[0]^3}{6} + (0.605F - 0.32R_B)0.3 - 0.4437F + 0.2667R_B$$

$$0 = -0.0045F + 0.1815F - 0.096R_B - 0.4437F + 0.2667R_B$$

$$0 = -0.2667F + 0.1707R_B \qquad R_B = \frac{0.2667F}{0.1707} = \frac{0.2667 \text{ x} 1922}{0.1707} = 3003\text{ N}$$

$R_A + F = R_B$ $R_A = 3003 - 1922 = 1081$ N

The deflection is hence $EIy = -F\frac{x^3}{6} + R_B\frac{[x-0.3]^3}{6} + (0.605F - 0.32R_B)x - 0.4437F + 0.2667R_B$

$$EIy = -1922\frac{x^3}{6} + 3003\frac{[x-0.3]^3}{6} + 201.85x - 51.9$$

When x = 0 the deflection is $y = \frac{-51.9}{EI}$ and this is the static deflection due to the point load.

$I = \pi D^4/64 = \pi(0.04^4)/64 = 1.257 \times 10^{-7}\ m^4$

$E = 200 \times 10^9$ $EI = 25130\ N\ m^2$

Static deflection = -2.065 mm

The natural frequency (discounting the distributed weight of the shaft is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{g}{y}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{2.065\,x10^{-3}}} = 10.97\ Hz$$

Alternate method

The strain energy induced in the shaft is E = ½ F y = ½ x 1922 x 2.065 x 10^{-3} = 1.9845 J

Kinetic energy = ½ M$\{\omega\ y\}^2$ = 1.9845 Mass = 195.92 kg

½ 195.92$\{\omega \times 2.065 \times 10^{-3}\}^2$ = 1.9845

$\omega \times 2.065 \times 10^{-3} = 0.14233$

$\omega = 68.92$ $f = \omega/2\pi = 10.97$ Hz

FREQUENCY OF SHAFT ALONE

$$\omega^2 = \frac{g\int y\,dx}{\int y^2 dx} \text{ where } y = \frac{-1922\frac{x^3}{6} + 3003\frac{[x-0.3]^3}{6} + 201.85x - 51.9}{EI}$$

$$\omega^2 = \frac{gEI\int\left(-1922\frac{x^3}{6} + 3003\frac{[x-0.3]^3}{6} + 201.85x - 51.9\right)dx}{\int\left(-1922\frac{x^3}{6} + 3003\frac{[x-0.3]^3}{6} + 201.85x - 51.9\right)^2 dx}$$

This is clearly far to difficult to do so an easier method is needed.