# D225 DYNAMICS OF MECHANICAL SYSTEMS Q4 2004

Tests conducted on a vacuum pump reveal that it produces a significant level of vertical vibration in the pump body with a frequency of 80 Hz. It is found that if the pump is supported very flexibly, this vibration has a r.m.s. acceleration magnitude of 12 $m/s^2$. The pump, which has a rigid body, has a total mass of 2 kg and the excitation can be treated as an external force acting on the pump body as illustrated in Figure (a).

In a particular application the pump is to be mounted on a piece of equipment which is very sensitive to vibration. The equipment may be regarded as being relatively rigid and heavy.

(a) If the pump is attached by a single spring as illustrated in Figure (b), determine the stiffness, k, of this spring such that the force transmitted to the equipment will not exceed 0.002 N r.m.s.

(b) Comment on practical considerations of this single-stage isolation.

(c) Figure (c) illustrates an alternative double-stage isolation system as a superior option. Calculate the force transmitted to the equipment with this arrangement.

(d) What alternatives would you suggest?

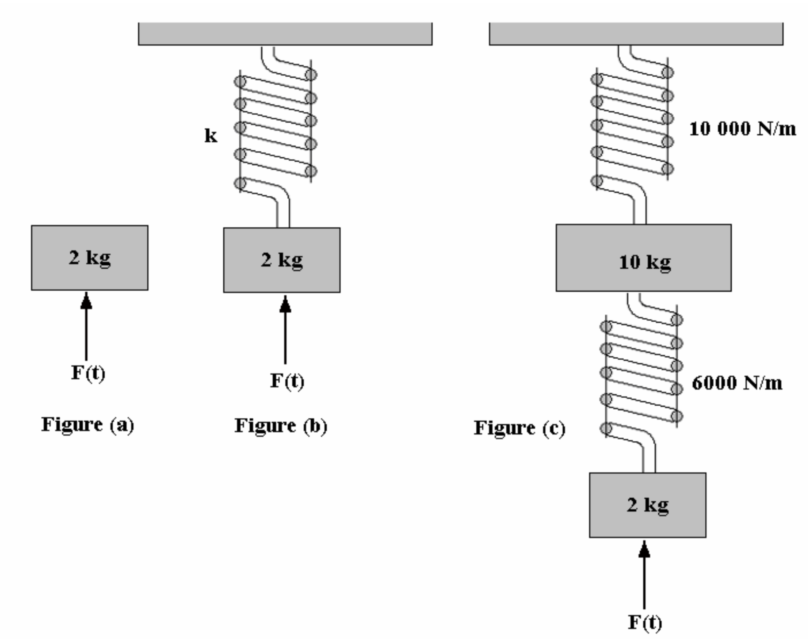


Figure (a) Figure (b) Figure (c)

SOLUTION

PRELIMINARY WORK

The question is not worded very clearly and difficult to interpret. Suppose that "very flexibly supported" means the spring force is negligible.

The applied force is $F_o \sin(\omega t)$ = The inertia force

$F_o \sin(\omega t) = m a$ This is a maximum when $\sin(\omega t) = 1$

The rms acceleration is 12 $m/s^2$ so the peak acceleration is $12\sqrt{2} = 16.97$ $m/s^2$

$F_o = m a = 2 \times 16.97 = 33.94$ N

The disturbing force is 33.94 sin ($\omega$t) and the rms value is $33.94\sqrt{2} = 24$ N

(a) The force transmitted to the support is $F_T = kA$

k = spring stiffness A = maximum spring deflection. $A = F_T/k$

The force applied to the mass = $F_o = kA - mA\omega^2$

(Derivation if $x = A\sin\omega t$ then $a = -A\omega^2 \sin\omega t$ so $F = ma = -mA\omega^2 \sin\omega t$ max value = $-mA\omega^2$)

$F_o = A(k - m\omega^2)$ and substitute $A = F_T/k$

$$\frac{F_T}{F_o} = \frac{k}{k - m\omega^2} = \frac{1}{1 - \dfrac{\omega^2}{m/k}} = \frac{1}{1 - \dfrac{\omega^2}{\omega_n^2}} = \frac{1}{1 - \dfrac{f^2}{f_n^2}}$$

$$\frac{F_T}{F_o} = T = \frac{1}{\left(1 - \dfrac{f^2}{f_n^2}\right)} = \frac{1}{\left(1 - r^2\right)} \qquad 1 - r^2 = \frac{1}{T} \qquad 1 - \frac{1}{T} = r^2 \qquad 1 - \frac{F_o}{F_T} = r^2$$

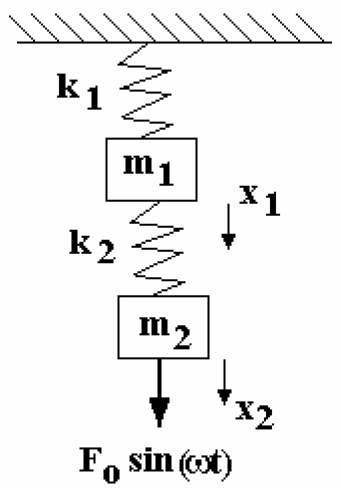
$$r^2 = \sqrt{\left(1 - \frac{F_o}{F_T}\right)^2} = \sqrt{\left(1 - \frac{24}{0.002}\right)^2} = \sqrt{(1 - 12000)^2} = 12000 \qquad r = 109.55$$

$$r = \frac{80}{f_n} = 109.55 \quad f_n = \frac{80}{109.55} = 0.73\text{ Hz}$$

$$f_n = 0.73\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad k = (0.73 \times 2\pi)^2 \times 2 = 42.1\text{ N/m}$$

(b) This system has a natural frequency well away from that at which the forcing occurs but will be prone to vibration as there is as no damping present and low frequency components of the forcing function may well make it vibrate. The spring rate is very low and the static deflection will be $x = (2 \times 9.81)/42.1 = 0.466$ m which is quite impractical.

(c)




The free body diagram shown yields the following force balance.

$m_1 a_1 + k_1 x_1 - k_2(x_2 - x_1) = 0$ and

$m_2 a_2 + k_2 (x_2 - x_1) = F_o \sin(\omega t)$

Change to the complex operator note that a is $d^2x/dt^2$

$-m_1 \omega^2 X_1 e^{j\omega t} + k_1 X_1 e^{j\omega t} - k_2(X_2 - X_1) e^{j\omega t} = 0$

$-m_1 \omega^2 X_1 + k_1 X_1 - k_2(X_2 - X_1) = 0$

$-m_1 \omega^2 X_1 + k_1 X_1 - k_2 X_2 + k_2 X_1 = 0$ ………………….(1)

$-m_2 \omega^2 X_2 e^{j\omega t} + k_2 (X_2 - X_1) e^{j\omega t} = F_o e^{j\omega t}$

$-m_2 \omega^2 X_2 + k_2 (X_2 - X_1) = F_o$

$-m_2 \omega^2 X_2 + k_2 X_2 - k_2 X_1 = F_o$………………….(2)

X is the amplitude of the motion.

From (1) we get $X_2 = X_1 \left[\dfrac{k_1 + k_2 - m_1\omega^2}{k_2}\right]$

Substitute into (2) $m_2\omega^2 X_1 \left[\dfrac{k_1 + k_2 - m_1\omega^2}{k_2}\right] + k_2 X_2 - k_2 X_1 = F_o$

$$X_1 \left[\frac{k_1 + k_2 - m_1\omega^2}{k_2}\right]\left[k_2 - m_2\omega^2\right] - k_2 X_1 = F_o$$

Noting that $k/m = \omega_n^2$

$$X_1 k_1\left[1+\frac{k_2}{k_1}-\frac{m_1}{k_1}\omega^2\right]\left[1-\frac{m_2}{k_2}\omega^2\right]-k_2 X_1 = F_o$$

$$X_1\left\{k_1\left[1+\frac{k_2}{k_1}-\frac{\omega^2}{\omega_1^2}\right]\left[1-\frac{\omega^2}{\omega_2^2}\right]-k_2\right\}=F_o$$

$\omega_1 = \sqrt{(10000/10)} = 31.623$ rad/s $\omega_2 = \sqrt{(6000/2)} = 54.772$ rad/s $\omega = 2\pi \times 80 = 502.65$ rad/s

Using peak values we get

$$X_1\left\{10000\left[1+\frac{6000}{10000}-\frac{502.65^2}{31.623^2}\right]\left[1-\frac{502.65^2}{54.772^2}\right]-6000\right\}=33.94\text{N}$$

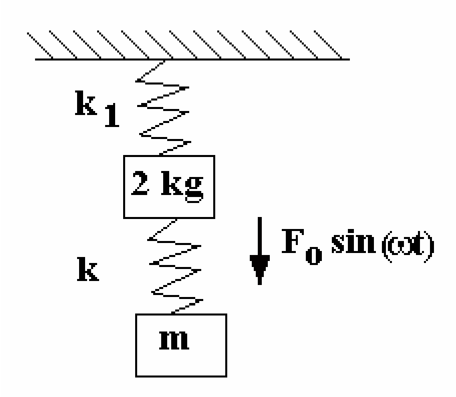
$$X_1\{[16252.6][-83.22]-6000\}=33.94\text{N}$$

$$X_1 = -\frac{33.94}{1358541} = -25 \times 10^{-6}\,\text{m}$$

The force transmitted is $k_1 X_1 = 10\,000 \times 25 \times 10^{-6} = 0.25$ N peak

This is larger than that transmitted previously.

(d) The answer is to detune the system as shown.




Without proof, when $\omega = \sqrt{(k/m)}$ the 2 kg mass will remain stationary.