## D225 DYNAMICS OF MECHANICAL SYSTEMS

## D225 2004 Q3

The diagram shows a simplified representation of the rotor assembly of a turbo-fan aircraft engine. The rotor can be modelled as a uniform cylinder 600 mm long and 400 mm diameter. The rotor mass is 100 kg with a centre of gravity 800 mm from the rear bearing, A. The fan can be modelled as a uniform (short) cylinder of length 100 mm and outside diameter 1.4 m (not drawn to scale). The fan has a mass of 100 kg with a centre of gravity 300 mm forward of the front bearing $B$.
(a) Compute the diametral and polar moments of inertia of the two cylinders at their respective centres of gravity.
(b) During take-off conditions, the rotor rotates at $7000 \mathrm{rev} / \mathrm{min}$ and the aircraft is pitching upwards with an angular velocity of $2 \mathrm{rad} / \mathrm{s}$ and an angular acceleration of 5 $\mathrm{rad} / \mathrm{s}^{-}$. Calculate the moments on the two cylinders due to the polar and diametral inertias. Assume zero angular velocity and angular acceleration in the roll and yaw directions.
(c) From part (b) determine the net reaction loads at the bearings if the net vertical acceleration of the rear bearing of this engine is 0.4 g at the instant in question.


## SOLUTION

The polar moment of inertia for a cylinder is $\mathrm{Jp}=\mathrm{MR}^{2} / 2$
For the rotor $\mathrm{Jpm}=100 \times 0.2^{2} / 2=2 \mathrm{~kg} \mathrm{~m}{ }^{2}$
For the fan $\mathrm{Jpf}=100 \times 0.7^{2} / 2=24.5 \mathrm{~kg} \mathrm{~m}^{2}$
The diametral moment of inertia is $(\mathrm{M} / 4)\left\{\mathrm{R}^{2}+\mathrm{h}^{2} / 3\right\}$
For the rotor $\mathrm{Jpm}=(100 / 4)\left\{0.2^{2}+0.6^{2} / 3\right)=4 \mathrm{~kg} \mathrm{~m}^{2}$
For the fan $\mathrm{Jpf}=(100 / 4)\left\{0.7^{2}+0.05^{2} / 3\right)=12.27 \mathrm{~kg} \mathrm{~m}^{2}$

