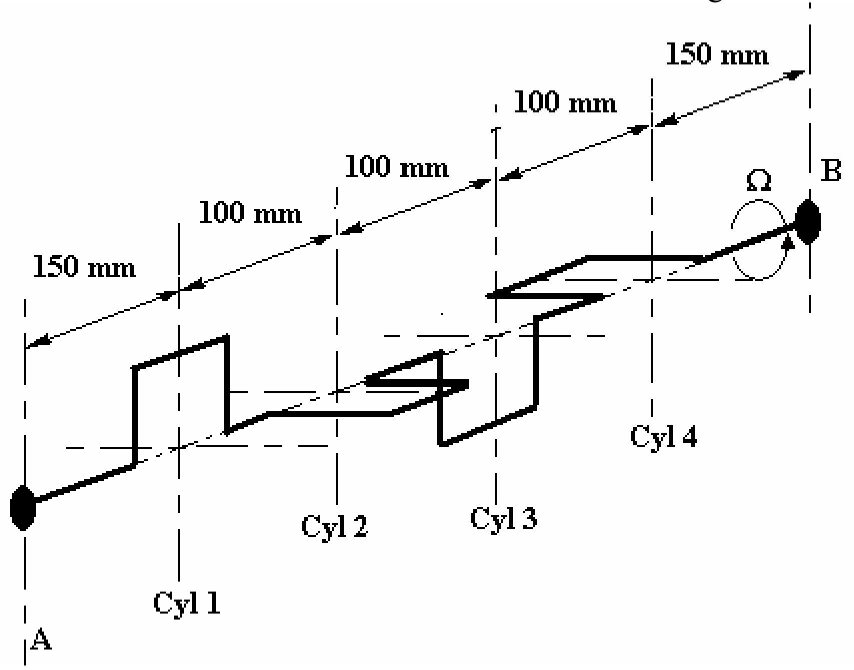
The diagram shows the crankshaft of a reciprocating compressor which has four vertical cylinders and the phasing of the cylinders is 1, 2, 3, 4 with uniform phase differences of 90° in all cases. The piston in each cylinder has a mass m of 400 g and a stroke 2R of 60 mm. The length L of the connecting-rod for each piston is 100 mm. The crankshaft is held in stiff bearings at ends A and B which are 600 mm apart and the centrelines of cylinders 1- 4 are 150 mm, 250 mm, 350 mm and 450 mm respectively from end A.

(a) Calculate the magnitudes and phasing of the primary and secondary unbalance forces at the bearings when the crankshaft is rotating at 3000 rev/min.

(b) In order to balance the compressor, two pairs of contra-rotating discs, to which balance weights can be attached, are fitted close to plane A and a further two pairs of contra-rotating wheels are fitted close to plane B. One pair at each end rotates at crankshaft speed,Ω and the second pair at each end rotates at twice crankshaft speed. Determine the imbalance (MR) which should be attached to the discs to achieve primary and secondary balance of the crankshaft.

(c) Could the primary components of unbalance due to the piston accelerations be offset by simply fitting deliberate unbalance to the crankshaft? Explain your response.

N.B. You may neglect the small distances between the discs and the bearings.



You may assume that the vertical acceleration of the pistons is given by $a = \Omega^2 R\left[\cos(\theta) + \frac{\cos(2\theta)}{n}\right]$

Where θ is the crankshaft angle and n = R/L

SOLUTION

The mass of the piston is M kg so the inertia force F produced is $F = M\Omega^2 R\left[\cos(\theta) + \frac{\cos(2\theta)}{n}\right]$

$M\omega^2 R$ is also the centrifugal force produced by a mass M rotating at radius R and angular velocity Ω.

The first term $F = M\Omega^2 R\cos(\theta)$ is called the PRIMARY FORCE.

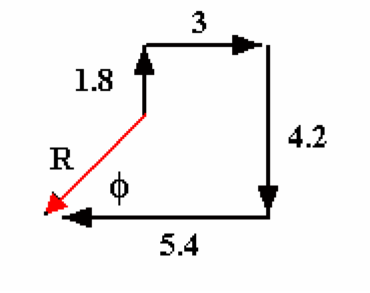
The second term $F = M\Omega^2 R\frac{\cos(2\theta)}{n}$ is called the SECONDARY FORCE.

$\Omega = 2\pi \times 500 = 100\pi$ rad/s R = 30 mm L = 100 mm n = 100/30=3.333 $\Omega = 100\pi$ rad/s

Both the primary and secondary forces are balanced as the value of MR is the same for each and the resultant is zero in both cases.

The primary turning moment about any reference plane is $M\Omega^2Rx \cos\theta$ where x is the distance from the reference plane. Taking the reference plane as plane A the turning moment for each cylinder

Cylinder	M (kg)	R (m)	MR (kg m)	x (m)	MRx (kg m^2)	Angle (deg)
1	0.4	0.03	12×10^{-3}	0.15	1.8×10^{-6}	0
2	0.4	0.03	12×10^{-3}	0.25	3×10^{-6}	90
3	0.4	0.03	12×10^{-3}	0.35	4.2×10^{-6}	180
4	0.4	0.03	12×10^{-3}	0.45	5.4×10^{-6}	270

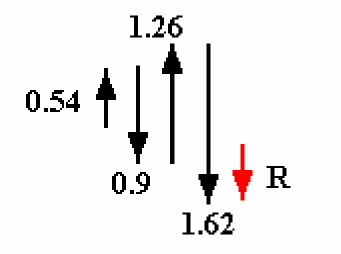


The resultant MRx is $\sqrt{\{(2.4 \times 10^{-3})^2 + (2.4 \times 10^{-3})^2\}} = 3.394 \times 10^{-3}$kg m^2
$\phi = 45^o$
Changing into turning moment we get TM = $(100\pi)^2 \times 3.394 \times 10^{-3}$ = 335 Nm
This is equivalent to a force at bearing B of 335/0.6 = 558 N
The phase angle is 225^o referred to cylinder 1.

The secondary turning moment about any reference plane is $TM = M\Omega^2Rx\dfrac{\cos(2\theta)}{n}$

Taking the reference plane as plane A the turning moment for each cylinder

Cylinder	M	R	MR	x	MRx/n	2θ
1	0.4	0.03	12×10^{-3}	0.15	0.54×10^{-3}	0
2	0.4	0.03	12×10^{-3}	0.25	0.9×10^{-3}	180
3	0.4	0.03	12×10^{-3}	0.35	1.26×10^{-3}	360
4	0.4	0.03	12×10^{-3}	0.45	1.62×10^{-3}	540

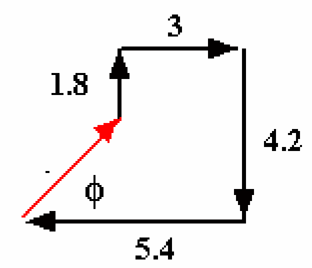


The resultant is 0.72×10^{-3} down.
Changing into turning moment we get TM = $(100\pi)^2 \times 0.72 \times 10^{-3}$ = 71 Nm
This is equivalent to a force at bearing B of 71/0.6 = 118 N
The phase angle is 180^o referred to cylinder 1.

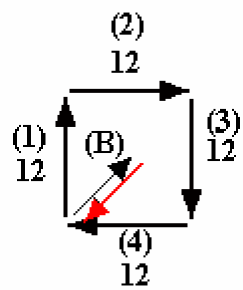
The forces on bearing A will be equal and opposite. Proof below but not needed in solution.

(b) To balance the primary moment we bring in the two balancing planes.

Cylinder	M (kg)	R (m)	MR (kg m)	x (m)	MRx (kg m^2)	Angle (deg)
A	M_A	0.03	0.03 M_A	0	0	
1	0.4	0.03	12×10^{-3}	0.15	1.8×10^{-6}	0
2	0.4	0.03	12×10^{-3}	0.25	3×10^{-6}	90
3	0.4	0.03	12×10^{-3}	0.35	4.2×10^{-6}	180
4	0.4	0.03	12×10^{-3}	0.45	5.4×10^{-6}	270
B	M_B	0.03	0.03 M_B	0.6	18 $M_B \times 10^{-3}$	



Draw the MRx polygon. The closing vector is 3.394×10^{-3} kg m^2 at 45^o as shown. This represents $18\ M_B \times 10^{-3}$ so $M_B = 0.188$ kg

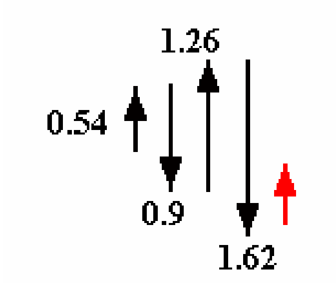


Evaluate 0.03 $M_B = 5.66 \times 10^{-3}$ and draw the MR polygon. The closing vector is 5.66×10^{-3}at 45^o as shown and this represents MR at A.
$5.66 \times 10^{-3} = 0.03\ M_A$ $\quad M_A = 0.188$ kg

For primary balance we would need a mass of 188/2 = 94 g on the primary discs set at 45^o either side of crank A on B and equal masses at opposite angles on discs A.

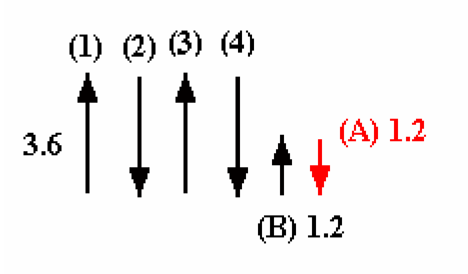
To balance the secondary terms we need to repeat the process with double angles. N= 3.333

Cylinder	M (kg)	R (m)	MR/n (kg m)	x (m)	MRx/n (kg m^2)	2θ (deg)
A	M_A	0.03	$9 \times 10^{-3} M_A$	0	0	
1	0.4	0.03	3.6×10^{-3}	0.15	0.54×10^{-3}	0
2	0.4	0.03	3.6×10^{-3}	0.25	0.9×10^{-3}	180
3	0.4	0.03	3.6×10^{-3}	0.35	1.26×10^{-3}	360
4	0.4	0.03	3.6×10^{-3}	0.45	1.62×10^{-3}	540
B	M_B	0.03	$9 \times 10^{-3} M_B$	0.6	$5.4\, M_B \times 10^{-3}$	



Draw the MRx/n polygon.
The closing vector is 0.72×10^{-3} at 0°. $5.4\, M_B \times 10^{-3} = 0.72 \times 10^{-3}$ $M_B = 0.133$ kg
Calculate $9 \times 10^{-3} M_B = 1.2 \times 10^{-3}$

Draw the MR/n polygon



The balancing vector is 1.2×10^{-3} at 180°.
$9 \times 10^{-3} M_A = 1.2 \times 10^{-3}$ $M_A = 0.133$ kg
Note that since the discs rotate at 2Ω rad/s the masses needed are first reduced by ¼ and then divided in 2 so the mass on each disc is $M_A/8$.
To balance the secondary components we need contra rotating masses on B of 133/8 = 16.6 g at 0° and the same on A at 180°

(c) The primary moment can balanced in the line of the cylinders with a single unbalanced mass but this induces forces normal to the line of the cylinder. Often a suitable compromise between the two components is made by changing the angle and the balance mass.