# D225 DYNAMICS OF MECHANICAL SYSTEMS Q1 2004

A design is being prepared for a space probe intended to land on a small planet and return to space carrying samples. The radius of the planet is 500 km. The gravitational pull of the planet is 1 $m/s^2$ at its surface and this gravitational pull is inversely proportional to the square of distance from its centre.

(a) Calculate the energy required to raise 1 kg of mass from the surface of the planet to a distance of 5000 km from its centre.
(b) Calculate the energy required to move the same 1 kg of mass a further 5000 km.
(c) If the planet has no atmosphere (and therefore no air-resistance), calculate the velocity a mass at the surface should have if it is to reach a distance of 5000 km from the centre of the planet without further propulsion.
(d) If the probe is to accelerate radially outward from the planet's surface with a constant acceleration of 50 $m/s^2$ for a period of 20 s, will the probe reach 5000 km from the planet centre without further propulsion?

**SOLUTION**

(a) d = distance from centre of planet. R = radius of planet. h = height above surface.
m = mass g = gravitational constant
Weight $w = mg$ $g = c/d^2$ when $d = 500 \times 10^3$ m $g = 1\ m/s^2$ so $c = 1 \times (500\,000)^2$
$g = (500\,000)^2/d^2$ $w = m\,(500\,000)^2/d^2$
Suppose the mass changes height by dh

The gravitational energy $= \int_0^h m\,g\,dh = \int_0^h \frac{m\,c\,dh}{d^2} = \int_0^h \frac{m\,c\,dh}{(R+h)^2}$

$h = 5 \times 10^6 - 500 \times 10^3 = 4.5 \times 10^6$ $R = 500 \times 10^3$

$$E = \int_0^h \frac{m\,c\,dh}{(R+h)^2} = \int_0^{4.5\times10^6} \frac{(500\times10^3)^2\,dh}{(500\times10^3+h)^2} = \left[\frac{(500\times10^3)^2}{(500\times10^3+h)}\right]_0^{4.5\times10^6}$$

E = – 50 000 + 500000 = 450000 Joules

(b) Moving up another $5 \times 10^6$ m the calculation becomes

$$E = \int_0^h \frac{m\,c\,dh}{(R+h)^2} = \int_{4.5\times10^6}^{9.5\times10^6} \frac{(500\times10^3)^2\,dh}{(500\times10^3+h)^2} = \left[\frac{(500\times10^3)^2}{(500\times10^3+h)}\right]_{4.5\times10^6}^{9.5\times10^6}$$

E = – 25 000 + 50000 = 25000 Joules

(c) The kinetic energy at lift off must equal the gravitational energy so
$mv^2/2 = 450000$ $v = 948.7$ m/s

(d) With constant acceleration the height reached will be $h = at^2/2 = 50 \times 20^2/2 = 10\,000$ m
The velocity reached will be $v = at = 50 \times 20 = 1000$ m/s
KE at this point is $mv^2/2 = 500\,000$ J
Energy required to reach d = 5000 km is :-

$$E = \left[\frac{(500\times10^3)^2}{(500\times10^3+h)}\right]_{10\times10^3}^{4.5\times10^6} = 440\,200 \text{ Joules}$$

Hence there is enough energy.