D225 2003 Q9

A differential gear box shown in the diagram combines the power of shafts (1) and (2) and delivers power at shaft 3. Each shaft carries a moment of inertia.

$J_1 = 2 \text{ kg m}^2$
$J_2 = 5 \times 10^{-4} \text{ kg m}^2$
$J_3 = 3 \text{ kg m}^2$

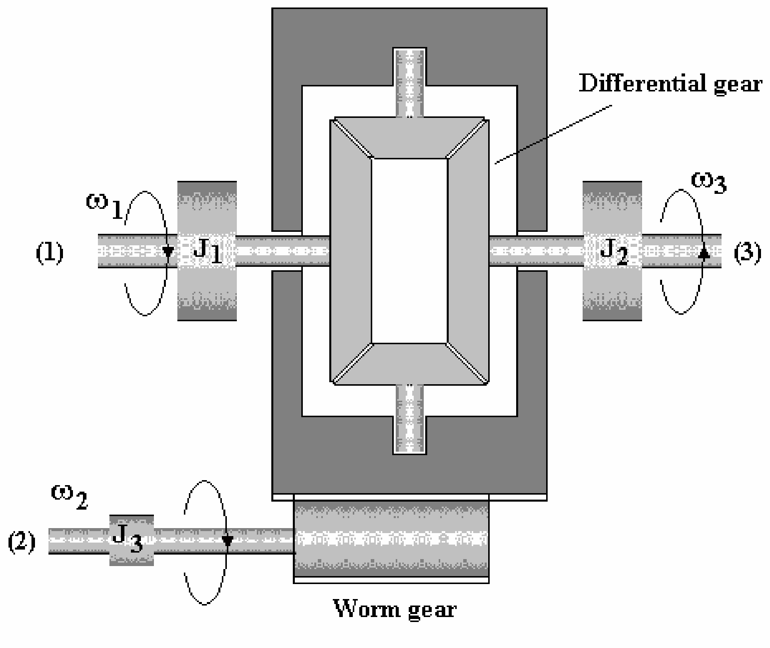
The speeds are related by the

$\omega_1 - \omega_2/20 - \omega_3 = 0$

The torques on each shaft are T_1, T_2 and T_3

The net power into the gear box is

$P_{net} = \omega_1 T_1 + \omega_2 T_2 + \omega_3 T_3$



(a) If there are no losses and the speeds are all constant, express T_1 and T_2 in terms of T_3 and show that the net power is zero.

(b) If 1 Nm of torque is applied at shaft (1) and nothing at the other two, calculate the angular acceleration of all the shafts.

SOLUTION
COMMENT – This question appears very odd and one wonders if the examiner understands what he is asking you do.

(a) It is obvious from the law of conservation of energy that if there is no loss and if the energy inside the gear box is constant, then the net power into the gear box must be zero so it is difficult to see what further proof is needed.

Examining the mechanics of the gear box, if shaft (2) was stationary, ω and ω_3 would be the same but reversed in direction.

If shaft (1) was stationary, shaft (3) would be rotated in the same direction as (2) but reduced by a gear ratio G
When shafts (1) and (2) are both rotated, shaft (3) rotates at $\omega_2/G - \omega_1$
$\omega_3 = \omega_2/G - \omega_1$ It would seem that G = 20
Given
$\omega_1 - \omega_2/20 - \omega_3 = 0$
$\omega_1 T_1 + \omega_2 T_2 + \omega_3 T_3 = 0$
It is not possible to specify T_1 nor T_2 entirely in terms of T_3

(b) If shaft (2) did not exist, the system would be a 1/1 gear box with reversal of direction and no resultant torque on the ring gear so there would be no tendency for the ring gear to rotate and hence no tendency for shaft to rotate. Shaft (2) would stay still and the inertia on shaft (3) would be accelerated. This being the case the problem is trivial and the inertia on (3) is surplus information. I doubt the examiner sees it this way. It appears to me that the speed of any one cannot be found without specifying the other two.

If the torques specified was on shaft (2) then we would have rotation of the other two and the question would make more sense.

If the inertial loads on (2) and (3) are referred to (1) we get:

$$T_2 = \alpha_1 \left[J_1 + J_2 \left(\frac{\omega_2}{\omega_1} \right)^2 + J_3 \left(\frac{\omega_3}{\omega_1} \right)^2 \right]$$ The effect of the gear ratio on inertia is squared.

If $T_2 = 1$ and as argued $\omega_2 = 0$ then $\omega_3 = \omega_1$ then

$$1 = \alpha_1 \left[2 + J_2 (0)^2 + 3(-1)^2 \right] \quad \alpha_1 = \frac{1}{5} = 0.2 \text{ rad/s}^2 \quad \alpha_2 = -0.2 \text{ rad/s}^2 \text{ (direction reversed)}$$