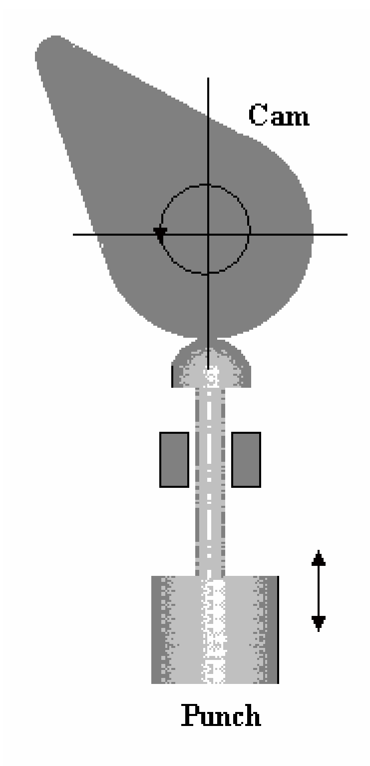
D225 2003 Q8

COMMENT – the original question appears to have been well dressed up to make a simple problem look much harder. Most of the data in the original question seems to be redundant. I think it is as simple as this but if you know different let me know.



A press for stamping out blanks uses a cam on a shaft with a flywheel running at 5 rev/s. The flywheel has a moment of inertia of 20 kg m^2.
200 J of energy are absorbed each time a blank is formed. The shaft is driven with a constant toque.

(a) Calculate the torque on the shaft.

SOLUTION
Energy absorbed per rev (2π radian) = 200 J = Angle x Torque T = $200/2\pi$ = 31.83 Nm

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(b) Determine the angular velocity at the start and finish of the operation.

SOLUTION
Assume the mean speed is 5 rev/s = 10π rad/s.
Max and min angular velocities are ω_1 and ω_2 respectively.
Coefficient of fluctuation of speed $\phi = (\omega_1 - \omega_2)/\omega$

The energy absorbed = energy put back in during the rest of the cycle so it follows that the fluctuation in energy is W = 200 J

$$\beta = \frac{\text{greatest fluctuation in kinetic energy}}{\text{work done}}$$

$$\beta = \frac{\frac{J\omega_2^2}{2} - \frac{J\omega_1^2}{2}}{W} = \frac{J}{2W}\left(\omega_2^2 - \omega_1^2\right) \text{Factorise the bracket}$$

$$\beta = \frac{J}{2W}\left(\omega_2 + \omega_1\right)\left(\omega_2 - \omega_1\right)$$

Since $\left(\omega_2 + \omega_1\right) = 2\omega$ and $\left(\omega_2 - \omega_1\right) = \varphi\omega$ then $\beta = \frac{J}{2W}(2\omega)(\omega\varphi)$

$\beta W = J\omega^2\varphi$
βW = greatest fluctuation in energy = 200 J

$\phi = \beta W/J_p\omega^2 = 200/\{20 \times (10\pi)^2\} = 200/19739 = 0.010132$

$(\omega_1 - \omega_2) = \omega\phi = 10\pi \times 0.010132 = 0.3183$ rad/s

The speed variation either side of the mean is 0.3183/2 = 0.1591 rad/s

$\omega_1 = 10\pi + 0.1591 = 31.575$ rad/s

$\omega_2 = 10\pi - 0.1591 = 31.257$ rad/s

(c) The punch has a mass of 25 kg. At the instant punching starts impulse is exchanged between the flywheel and punch. The punch velocity is related to the rotation of the cam by v = 0.001 Ω where Ω is degrees per second. Calculate the fraction of the speed lost at this instant.

SOLUTION

The examiner mentions referring a mass to a rotating inertia. The following is my uninformed first guess at the method and should be treated with care as it is certainly not correct.

$\Omega = 5 \times 360 = 1800$ deg/s $\omega = 1800 \times \pi/180 = 10\pi$ rad/s
$v = 0.001\Omega = 1.8$ m/s
Normally $v = \omega R$ where R is the radius $= v/\omega = 1.8/10\pi = 0.0573$ m
We might guess that punch is effectively operated by a force at a radius of 0.0573 m but this does not appear to be needed.

Impulsive force = M dv/dt
Impulsive torque = $J \Delta\omega/t = F R$ so the impulsive force at radius R is $(J/R) \Delta\omega/t$

Equate forces $(J/R) \Delta\omega = M \Delta v$

$\Delta\omega = R M \Delta v/J$

$\omega_1 - \omega_2 = R \times 25 \times \Delta v/20$

Δv is from 0 to v_2 and v_2 corresponds to $\omega_2 R$

$\omega_1 - \omega_2 = R \times 25 \times \omega_2/20R$

$\omega_1 - \omega_2 = 25 \times \omega_2/20$

$\omega_1 = 1.25\omega_2$ $\omega_2 = 0.8\, \omega_1$