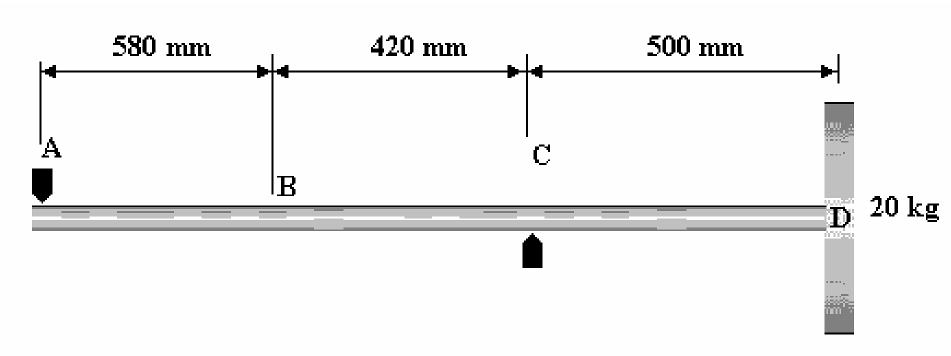
D225 2003 Q7

Comment – I am not at all sure what the examiner expects in the way of a solution. My solution given below would take a very long time to do and needs a good knowledge of beam and strain energy theory. In particular the affect of the disc rotation about the diameter is new to me. If anyone knows a better way, please let me know.

A disc is mounted on a shaft as shown. It has a second moment of inertia about a diameter of 0.1 kg m^2. A and B are two simple supports. The shaft deflects upwards 0.31 mm at point B with no rotation. It deflects downwards 1.2 mm at D with a rotation of 2.8×10^{-3} radian.



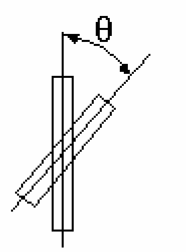
(a) Using Rayleigh's method, determine the first natural frequency of transverse vibration ignoring the mass of the shaft.

An approximate solution based on static deflection of the mass is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \text{ and } k = \text{weight/static deflection} \quad k = Mg/x_s$$

$$\text{Substituting we have } f = \frac{1}{2\pi}\sqrt{\frac{g}{x_s}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.0012}} = 14.39 \text{ Hz}$$

This is a result of Rayleigh's method anyway but does not take into account the rotation of the disc about the diameter.



Rayleigh's method basically states that the maximum strain energy in the elastic element is equal to the maximum kinetic energy of the oscillating system. The strain energy is easily calculated based on the static deflection of the shaft. Ignoring the mass of the shaft the disc on the end has deflected down 1.2 mm and rotated 2.8×10^{-3} radian. The static rotation does not involve any energy but the vertical deflection is the result of the stressing in the shaft. It can be safely assumed that deflection is proportional to force.

The work done by a spring force is ½ Fy_m
The force causing the deflection is the weight of the disc Mg so Work = ½ Mgy_m
Work done = strain energy stored so U = ½ Mgy_m

Suppose the motion up and down is given as $y = y_m \sin \omega t$
The velocity up and down is $v = \omega y_m \cos \omega_n t$ and the maximum value is $\omega_n y_m$
Max kinetic energy = ½ $M/\{\omega_n y_m\}^2$

Suppose the rotation of the disc about a diameter is $\theta = \theta_m \sin \omega t$
The angular velocity is $\omega^* = \theta_m \omega_n \cos \omega_n t$ and this has a maximum of $\theta_m \omega_n$
The maximum kinetic energy is ½ $I_d \{\omega_n \theta_{max}\}$
Equate to the strain energy
½ Mgy_m =½ $M/\{\omega_n y_m\}^2$ + ½ $I_d \{\omega_n \theta_{max}\}$

We can use any corresponding values of deflection and rotation.

Put y_m= 0.0012 m and θ_m =2.8 x 10^{-3} rad and
$\frac{1}{2}$ 20 x 9.81 x 0.0012 = $\frac{1}{2}$ 20 ω_n^2 $(0.0012)^2$ + 0.05ω_n^2 $(2.8 \times 10^{-3})^2$ = 14.792 x $10^{-6}\omega_n^2$
0.11772 = 14.792 x 10^{-6} ω_n^2
ω_n^2 = 7.958 x 10^3 ω_n = 89.21 rad/s f_n = 14.198 Hz which is close to first prediction.

(b) The affect of the mass of the shaft will be to reduce the frequency. If the frequency found is f_1 and the frequency of the shaft with no disc is f_2 then the resultant frequency is given by Dunkerley as

$1/f_n^2 = 1/f_1^2 + 1/f_2^2$ so it follows that f_n is reduced.

(c) The frequency of the shaft with no disc would be found from $\omega^2 = \frac{g\int y_m dx}{\int y_m^2 dx}$ and this would involve an even longer piece of work to solve. There must be some approximate method using the lumped mass. From A to C the shaft is raised and from C to D it is lowered. If we could estimate the means we could include this in the strain energy.