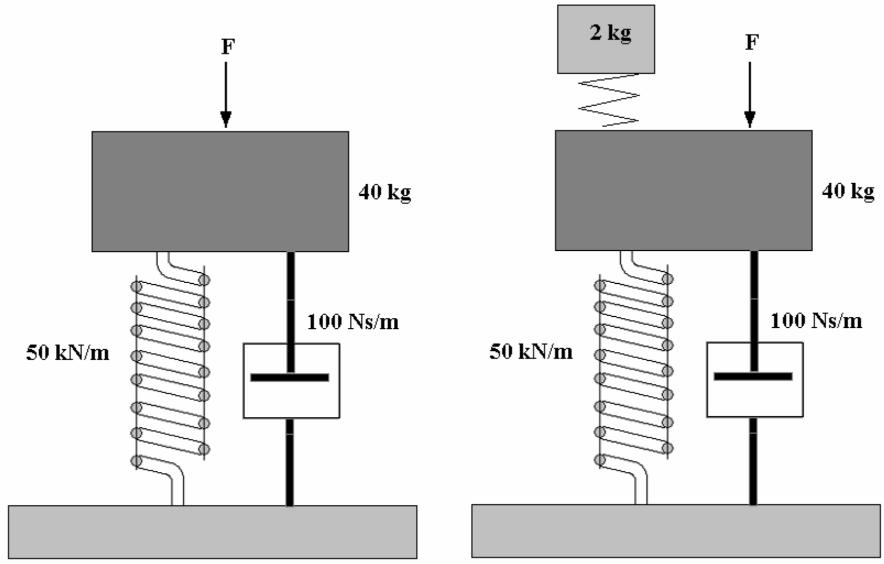
A concrete block of mass 40 kg is mounted on a flexible support which in turn is connected to a ship deck. The support is a spring of stiffness 50 kN/m and damping coefficient 100 Ns/m. A force is applied to the support as shown (fig a).

(a) Calculate the undamped natural frequency, the damped natural frequency, the damping factor and the Q ratio.
(b) Develop an expression for the transmissibility factor between the system and the deck given $F = 100\sin(\omega t)$.
(c) The frequency of the applied force is between 30 and 31 rad/s. A vibration absorber is placed on top of the block (figure b) in the form of a mass of 2 kg and a spring of stiffness k. Calculate a suitable value for k and estimate the new transmissibility factor at $\omega = 31$ rad/s.



(a) (b)

SOLUTION

(a) Data $M = 40$ kg $k = 50 \times 10^3$ N/m $c = 100$ Ns/m

Natural Frequency of System $\omega_n = \sqrt{(k/M)} = 35.355$ rad/s $f_n = \omega_n/2\pi = 5.627$ Hz
$Cc = \sqrt{(4 M k)} = 2828$ Ns/m $\delta = c/Cc = 0.035$
The damped frequency is $\omega_d = \omega_n\sqrt{(1- \delta^2)} = 35.333$ $f_n = 5.623$ Hz

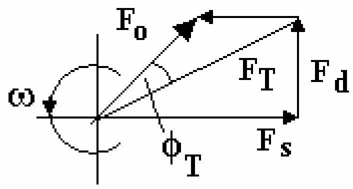
Magnification Factor applies when a sinusoidal disturbing force is applied.

$$MF = \sqrt{\frac{1}{\left(1 - r^2\right)^2 + (2\delta r)^2}} \qquad r = \omega/\omega_n$$

This has a peak value when $\omega/\omega_n = \sqrt{(1 - \delta^2)} = 0.999$ hence MF = 14.151 (Max)

(b) **TRANSMISSIBILITY**

When a mass vibrates on an elastic support, a force is transmitted through the spring and damper to the frame or ground. This is the sum of the spring and damping force. This may be illustrated with the vector diagram.



From the vector diagram we deduce that the transmitted force is

$F_T = \sqrt{(F_s^2 + F_d^2)}$

$F_s = kA$ and $F_d = cA\omega$ $\quad F_T = \sqrt{\{(kA)^2 + (cA\omega)\}}$

The ratio F_T/F_o is called the transmissibility ratio. In this case $F_o = 100$ N

$$\text{transmissibility} = \frac{\sqrt{(kA)^2 + cA\omega}}{100}$$

(c) Without derivation (see tutorial) a fact worth memorising is that for this system, the detuned frequency is the natural frequency of the second mass – spring system.

and $\dfrac{x_1}{F_o} = 0$ when $\omega = \sqrt{(k_2/M_2)}$

$k_2 = \omega^2 M_2$

Taking $\omega = 30.5$ we get $k_2 = (30.5)^2 \times 2 = 1860.5$ N/m