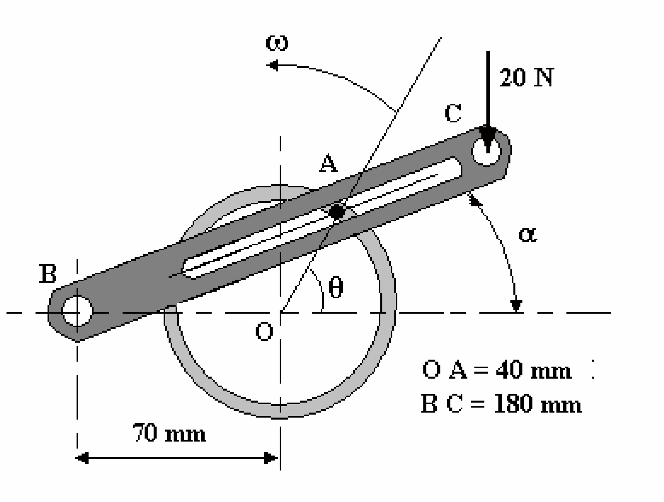
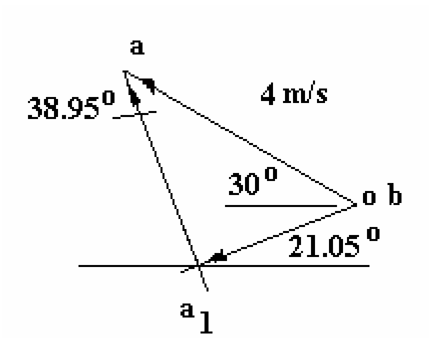
D225 2003 Q3

The diagram shows part of a quick return mechanism. The pin A slides in the slot when the disc is rotated.

(a) Calculate the angular velocity and acceleration of link BC when θ = 60o and ω = 100 rad/s. (α is 21.052° at this position)
(b) The link BC has a mass of 0.8 kg distributed along the length and a mass of 0.4 kg at C. In addition of vertical force of 20 N acts at C. Calculate the torque input to the disc at the instant shown.
The moment of inertia of a uniform rod about its centre of gravity is $ml^2/12$

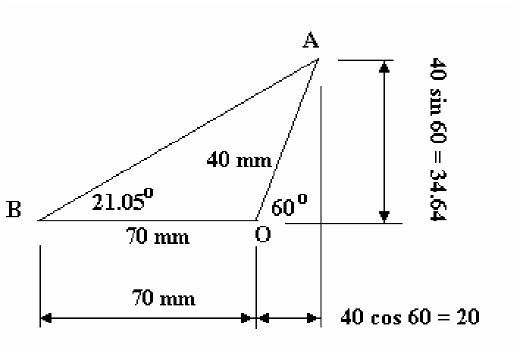


(a) The tangential velocity of A relative to O is ωR = 100 x 0.04 = 4 m/s. The velocity diagram is constructed as shown.



The tangential velocity of pin A relative to B is
$(V_{A1})_B = a - a_1 = 4\cos(38.95^\circ) = 3.11$ m/s
The radial velocity of A relative to B is
$(V_A)_B = 4\sin(38.95^\circ) = 2.515$ m/s

The length of BA is easily calculated from the diagram.



$BA = \sqrt{(34.64^2 + 90^2)} = 96.44$ mm

The angular velocity link BC = 3.11/BA = 32.2 rad/s
(This checks with a computer solution)

ANALYTICAL METHOD

The angle of link BC is

$$\alpha = \tan^{-1}\left[\frac{40\sin\theta}{70 + 40\cos\theta}\right] = \tan^{-1}\left[\frac{\sin\theta}{7/4 + \cos\theta}\right]$$

The angular velocity is dα/dt and the tools for doing the differentiation are given in the question as follows.

$$\text{Let } x = \left[\frac{\sin\theta}{7/4 + \cos\theta}\right] \quad \frac{d\alpha}{dx} = \frac{1}{1+x^2} = \frac{1}{1 + \left(\frac{\sin\theta}{7/4 + \cos\theta}\right)^2} \quad \frac{dx}{d\theta} = \frac{1 + 7/4\cos\theta}{(7/4 + \cos\theta)^2}$$

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dx}\frac{dx}{d\theta} = \frac{1}{1 + \left(\frac{\sin\theta}{7/4 + \cos\theta}\right)^2}\,\frac{1 + 7/4\cos\theta}{(7/4 + \cos\theta)^2}$$ put $\theta = 60^\circ$ and evaluate

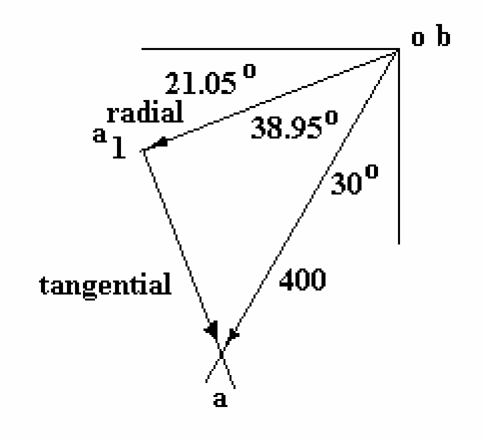
dα/dθ = 0.316

θ = ω t so dθ = ω dt dt = dθ/ω so dα/dt = 0.316 x ω = 31.6 rad/s which is close to the answer found before.

Next construct the acceleration diagram.

Link O A only has centripetal acceleration inwards

$(a_A)_O = \omega^2 R = 100^2 \times 0.04 = 400 \text{ m/s}^2$



The pin A has a tangential acceleration and Coriolis acceleration normal to the link. It has centripetal acceleration and radial acceleration towards the centre of rotation. B. The diagram can be constructed without calculating them.
The Corioilis acceleration is 2 ω v where ω = 32.2 and v is the radial velocity = 2.515 m/s
The Coriolis term is hence 161.97 m/s^2

The tangential acceleration of A relative to B is
$a_1 - a = 400 \sin 38.95 = 251.46 \text{ m/s}^2$
Part of this is the Coriolis so the tangential acceleration is 251.46 - 161.97 = 89.49 m/s^2
The angular acceleration of link AC is $\alpha = 89.49 /BA = 89.49/0.09644 = 928 \text{ rad/s}^2$

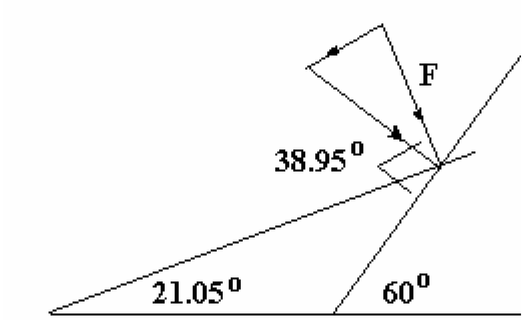
The direction is negative (clockwise) so it is decelerating.

(This answer checks out with the computer answer but I would not attempt a second integration)

(b) Calculate the force acting on the pin normal to the link BC. Take the centre of gravity at the mid point.

J (link) = $0.8 \times 0.18^2/12 = 0.00216 \text{ kg m}^2$ about the centre of gravity which is 90 mm from B.
About point B, J(link) = $0.00216 + 0.8 \times 0.09^2 = 0.00864 \text{ kg m}^2$ (or $ml^2/3 = 0.00864$).
The mass at point C produces a moment of inertia at B of $0.4 \times 0.18^2 = 0.01296 \text{ kg m}^2$
The total about C is J = 0.00864 + 0.01296 = 0.0216 kg m^2
The torque needed on the link to accelerate it is $T = J\, d\alpha/dt = 0.0216 \times (-928) = -20$ Nm

In addition a torque is needed to overcome the 20 N. The component of the 20 N acting normal to B C is 20 sin 21.05° = 7.184 N acting at 0.18 m from B.
The torque required on the link is
T =7.184 x 0.18 = 1.293 Nm



Total torque required about B is
T = -20 + 1.293 = -18.7 Nm

The pin A exerts a force at radius of 96.44 mm.
F = T/AB =-18.7/0.09644 = -194 N

Now resolve this force normal to OA and get
F = -194 cos 38.95° = -150.8 N

The torque required on disc C about O is T = -150.8 x 0.04 = -6.03 Nm