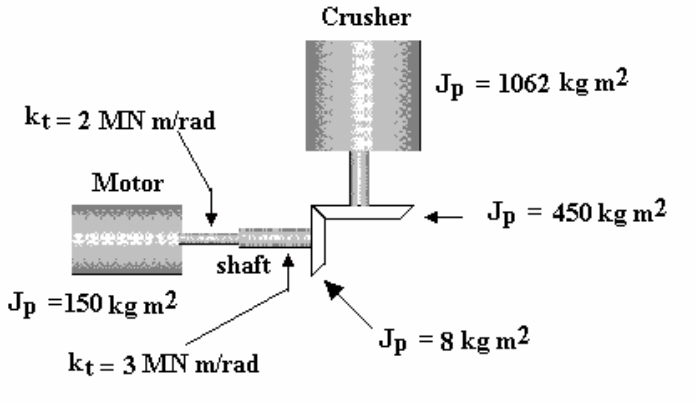
D225 2003 Q2

A motor drives a crusher through a 6/1 reduction gear as shown. The polar moment of inertia of each relevant part is shown. The motor shaft is in two parts with torsional stiffness as shown.



(a) Calculate the effective moment of inertia at the motor.

SOLUTION
The moment of inertia on the vertical shaft is 1062 + 450 = 1512 kg m^2
On the motor shaft this equivalent to 158 /(gear ratio)^2 = 1512/6^2 = 42 kg m^2
Note the effect is reduced because the gears reduce the speed of the crusher.
The total effective moment of inertia experienced by the motor is 150 + 8 + 42 = 200 kg m^2

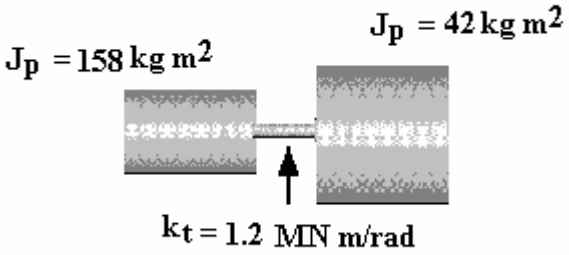
(b) Calculate the equivalent torsional stiffness of the shaft.

SOLUTION
$k_t = T/\theta = GJ/L$ The same torque on both lengths.
$k_{t1} = T/\theta_1 = GJ_1/L_1$ $k_{t2} = T/\theta_2 = GJ_2/L_2$

$\theta_1 = T L_1 /GJ_1$ $\theta_2 = T L_2 /GJ_2$
$\theta = \theta_1 + \theta_2 = T L_1 /GJ_1 + T L_2 /GJ_2 = T\{ L_1 /GJ_1 + L_2 /GJ_2\} = T\{1/ k_{t1} + 1/ k_{t2} \}$
$\theta/T = 1/ k_t = \{1/ k_{t1} + 1/ k_{t2}\}$
$k_t = (k_{t1} k_{t2})/(k_{t1} + k_{t2}) = (2 \times 3)/(2+3) = 1.2$ MN m/rad

(c) The equivalent system has a torsional mode of oscillation at 0 Hz and one other resonance. Calculate the second frequency.

SOLUTION
The system is to be treated as two inertias.



$$\omega_n^2 = k_t \frac{J_1 + J_2}{J_1 J_2} = 1.2 \times 10^6 \frac{158 + 42}{158 \times 42} = 36166 \quad \omega = 190.17 \text{ rad/s} \quad \text{or } 30.27 \text{ Hz}$$