Q7 2000

A thin circular disk of radius r is rotating about its z axis with angular velocity p. The yoke in which it is mounted rotates about the Z axis at angular velocity ω_1.

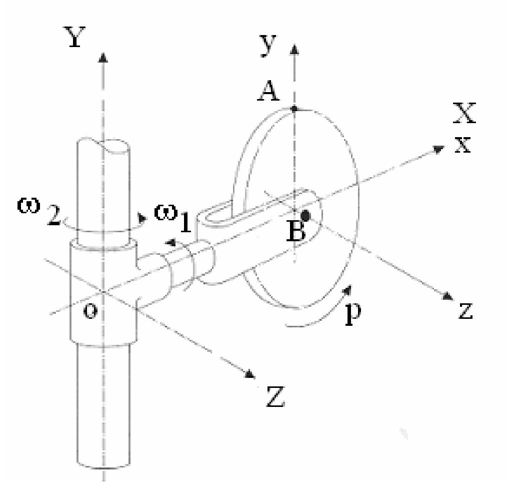
At the same time the whole assembly rotates about the axis O Y with constant angular velocity ω_2.

(a) Calculate the absolute velocity of point A at the instant shown.

(b) Calculate the absolute acceleration of point A at the instant shown.

Data Disk radius r = 0.05 m OB = 0.1 m

Disk spin rate p = 5 rev/s ω_1 = 2 rev/s ω_2 = 1 rev/s



SOLUTION

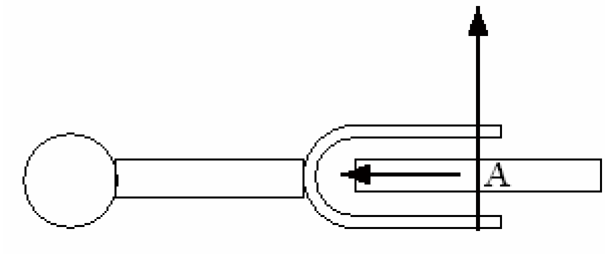
I hope there is not some deeper meaning to this question concerning gyroscopes. Otherwise it seems a simple question off adding vectors. The choice of symbols is not very good as ω is normally used for rad/s.

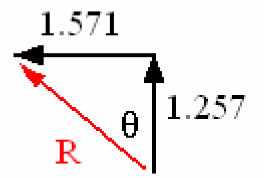
(a) Point A has a velocity in the x direction of $-2\pi p\, r = -2\pi\,(5)(0.05) = -1.571$ m/s

Point A has a velocity in the z direction of $-2\pi\omega_1\, r = -2\pi\,(2)(0.05) = -0.628$ m/s

Point A has a velocity in the z direction of $-2\pi\omega_2\,(oB) = 2\pi\,(1)(0.1) = -0.628$ m/s

Adding these up we have a total velocity in the z direction of -1.257 m/s

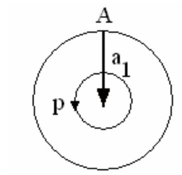




The resulting velocity is $v = (1.571^2 + 1.257^2)^{1/2} = 2.01$ m/s

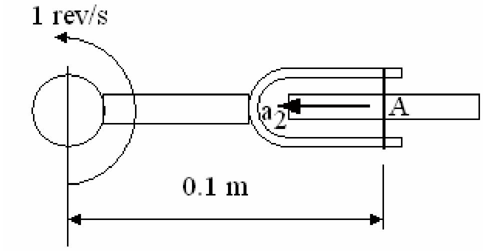
$\theta = \tan^{-1}(1.571/1.257) = 51.3^{\circ}$

(b) The only accelerations present are centripetal (ang vel)^2 x radius



Point A has a centripetal acceleration as shown.

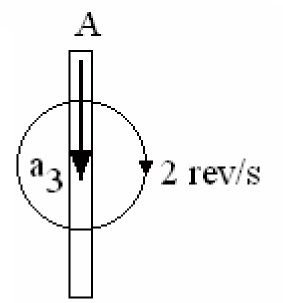
$a_1 = (2\pi p)^2 \text{ x } r = (2\pi\ 5)^2 \text{ x } 0.05 = 49.35 \text{ m/s}^2$



Point A has another centripetal acceleration as shown.

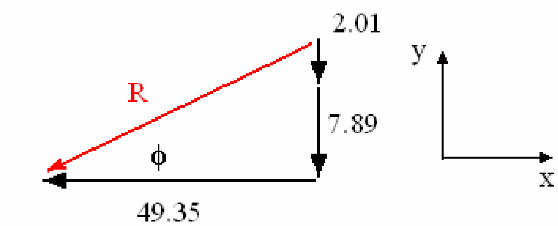
$a_2 = (2\pi\ 1)^2 \text{ x } 0.1 = 3.95 \text{ m/s}^2$

Point A has a third centripetal acceleration as shown.



$a_3 = (2\pi\ 2)^2 \text{ x } 0.05 = 7.89 \text{ m/s}^2$

Now add the vectors



Resultant acceleration $a = (9.9^2 + 49.35^2)^{1/2}$

$A = 50.33 \text{ m/s}^2$

$\phi = \tan^{-1}(7.89/49.35) = 9.2^{\circ}$

Direction as indicated.