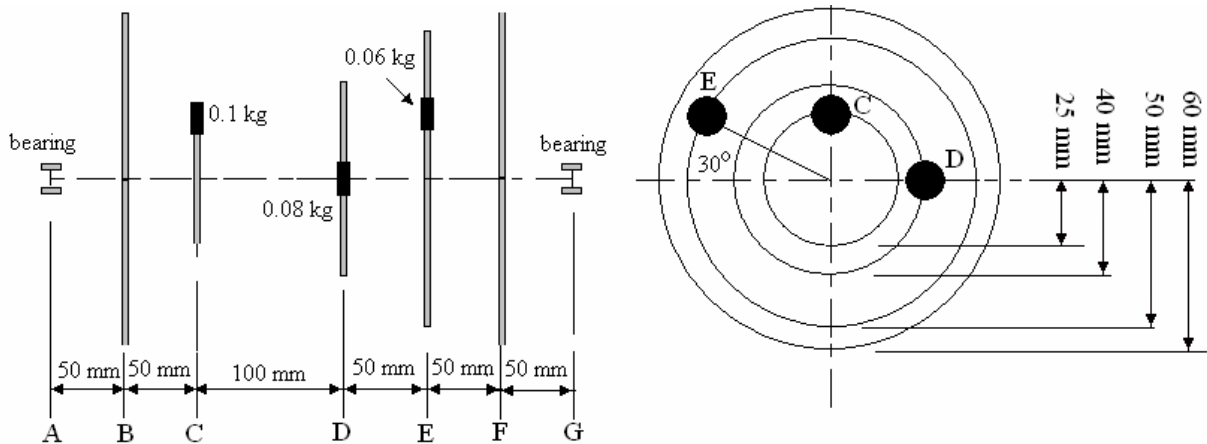


Dynamics of Mechanical Systems Q2 2000

The rigid rotor shown from a machine has unbalanced masses on disks C, D and E as indicated. Discs B and F are to be used for balancing. The shaft rotates at 1000 rev/min.

- Calculate the rotating forces on the bearings before balancing.
- Determine the masses and their angular position that must be placed on B and F at a radius of 60 mm in order to produce complete balance.



SOLUTION

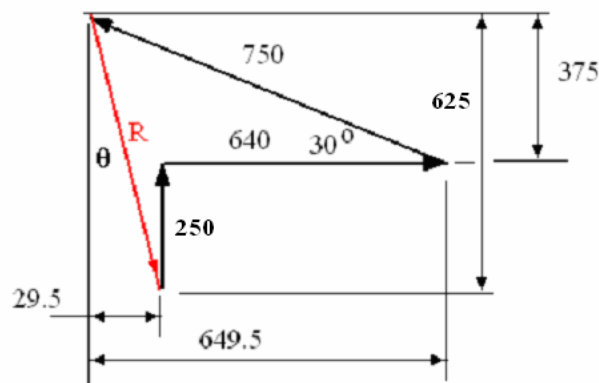
Angular velocity $\omega = 2\pi N/60 = 2\pi (1000)/60 = 209.4 \text{ rad/s}$

(i)

Use the tabular method. Make A the reference plane. X is measured from B
Remember centrifugal force = $M \omega^2 r$ and the moment about the ref plane is $M \omega^2 r x$
First find the out of balance moment.

Plane	Mass	Radius	x	Mr	Mrx
C	0.1	25	100	2.5	250
D	0.08	40	200	3.2	640
E	0.06	50	250	3.0	750

These are vectors and we must find the resultant Mrx vector. Draw vector diagram and produce the resultant as shown.

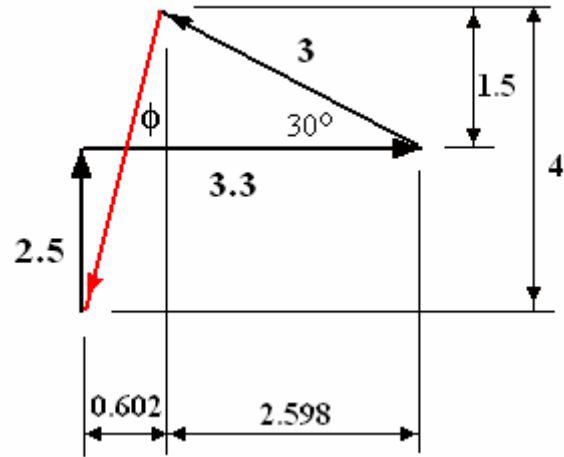


$$R = (29.5^2 + 625^2)^{1/2} = 625.7 \text{ kg mm}^2 \quad \theta = \tan^{-1}(29.5/625) = 2.7^\circ$$

The out of balance moment is $\omega^2 \times 625.7 \times 10^{-6} = 209.4^2 \times 625.7 \times 10^{-6} = 27.44 \text{ N m}$ based on plane A.

The force on bearing G that produces an equal and opposite couple is $27.44/0.35 = 78.4 \text{ N}$

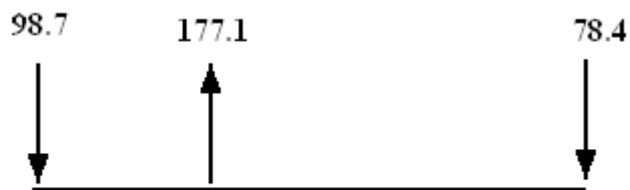
We need the resultant force so we must draw the MR polygon.



The total Mr is $(0.602^2 + 4^2)^{1/2} = 4.04$ kg mm.

The out of balance force $F = 4.04 \times 10^{-3} \times 209.4^2 = 177.1$ N

The force on bearing A is $177.1 - 78.4 = 98.7$ N in the same direction.

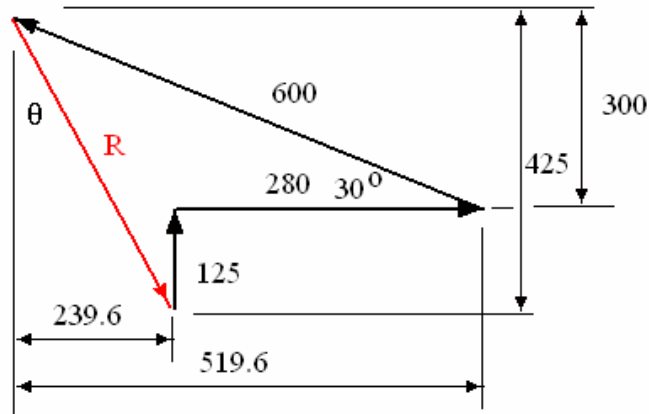


angle shown with forces vertical

(ii) Take B as the reference plane

Plane	Mass	Radius	x	Mr	Mrx
B	M_B	60	0	$60M_B$	0
C	0.1	25	50	2.5	125
D	0.08	40	150	3.2	480
E	0.06	50	200	3.0	600
F	M_F	60	250	$60M_F$	$15000 M_F$

Draw the MRx vector diagram.

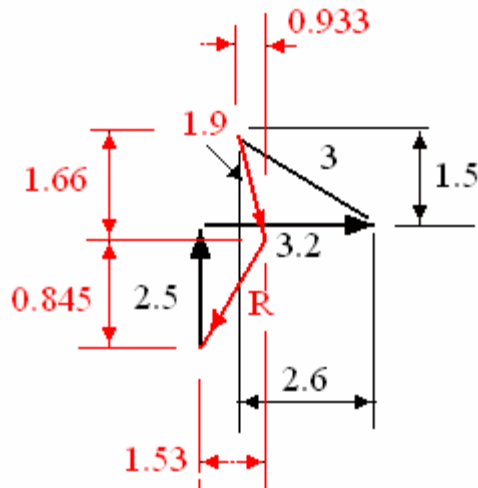


For complete balance R must be the MRx value for the mass on disk F.

$$R = (239.6^2 + 425^2)^{1/2} = 487.9 \text{ kg mm}^2 \quad \theta = \tan^{-1}(239.6/425) = 29.4^\circ$$

It follows that for complete balance $15000 M_F = 487.9 \text{ kg mm}^2$
 $M_F = 487.9/15000 = 0.0325 \text{ kg}$

Now draw the MR vectors. Evaluate $60 M_F = 1.952$



$$R = (1.53^2 + 0.845^2)^{1/2} = 1.75 \text{ kg mm}$$

This represents the disc B so $60 M_B = 1.75 \text{ kg mm}$

$$M_B = 0.0292 \text{ kg}$$

The angle for mass B is $\tan^{-1}(1.53/0.845) = 61^\circ$ left of vertical down.

