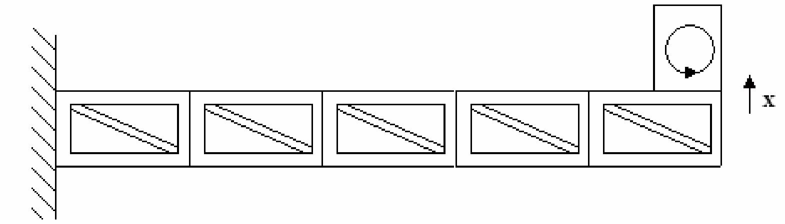
DYNAMICS OF MECHANICAL SYSTEMS Q9 1998

The tail rotor of a helicopter can be modelled as a cantilever beam with a mass at the free end representing the gear box and rotor. The combined mass is 50 kg and the mass of the beam is negligible. The natural frequency of the system is 8 Hz and the log decrement δ is 0.953. The mass of a single rotor blade is 4 kg with a centre of gravity 0.15 m from the centre of the rotor axis.



(a) Determine the damping ratio ξ for the system.
(b) Determine the stiffness k for the beam.
(c) Calculate the amplitude of response of the tail rotor if while rotating at 1200 rev/min one of the blades breaks off and becomes detached.
Data - If a force F cos(ωt) acts on the mass m of a single degree of freedom system with stiffness k and viscous damping coefficient c. the steady state displacement is given by :

$$x = (F/k)[(1 - r^2)^2 + (2\xi r^2)^2]^{-\frac{1}{2}}\cos(\omega t - \phi)$$

$r = \omega/\omega_n$ $\quad \omega_n = (k/m)^{\frac{1}{2}}$ $\quad \xi = c/2(mk)^{\frac{1}{2}}$ $\quad \phi = \arctan[2\xi r/(1 - r^2)]$

(a) The log decrement is $\quad \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\delta^2}}$

$$0.953 = \frac{2\pi\xi}{\sqrt{1-\delta^2}} \qquad \frac{\xi}{\sqrt{1-\xi^2}} = 0.15167 \qquad \frac{\xi^2}{1-\xi^2} = 0.023$$

$$\xi^2 = 0.023 - 0.023\xi^2 \qquad 1.023\,\xi^2 = 0.023 \qquad \xi = \sqrt{0.02248} = 0.15$$

(b) $\quad f_n = 8$ Hz $\quad \omega_n = 2\pi \times 8 = 50.265$ rad/s

$$50.265 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{50}} \qquad 2526.6 = \frac{k}{50} \qquad 126.33 \times 10^3\ \text{N/m} = k$$

(c) If a tail rotor breaks of the mass is reduced to 46 kg and the out of balance force is equal and opposite to the centripetal force that formerly existed.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{126330}{46}} = 52.405\ \text{rad/s}$$

$\omega = 2\pi(1200/60) = 125.66$ rad/s
$F = M\omega^2 r = 5 \times 125.66^2 \times 0.15 = 11843.5$ N

If this only acts in the vertical plane then the force in the vertical direction is :
$F = 11843.5\cos(\omega t - \phi)$ where ϕ is an arbitrary starting angle.

From the equation for displacement x it is clear that the amplitude is given by:
$r = \omega/\omega_n = 125.66/52.405 = 2.4$ (Note if the mass is 50 kg, r = 2.5 which may be what the examiner designed into the solution)

$$A = \frac{F}{k}\left[(1-r^2)^2 + (2\,\xi\, r^2)^2\right]^{-\frac{1}{2}} = \frac{11843.5}{126330}\left[(1-2.4^2)^2 + (2 \times 0.15 \times 2.4^2)^2\right]^{-\frac{1}{2}}$$

$$A = 0.09375[22.66 + 2.986]^{-\frac{1}{2}}$$

$$A = 0.0185\ \text{m}$$